CRASH ANALYSIS OF SELECTED ROADWAY SEGMENTS IN PALM BAY

Prepared by:

Kwabena Ofosu, Growth Management Department

Elia Twigg, PE, Division Manager, Public Works Department

Alan Done, Traffic Operations Division, Public Works Department

Lori Parker, Right-of-Way Services Division Manager, Public Works Department

5240 Babcock St, NE

Palm Bay, FL 32905

November 25, 2009

Signed: ___________________________ PE No.:____________

John G. Rodgers

Summary

Staff was asked to analyze crashes on selected City roadway segments to enable determinations be made regarding the City streetlight program. Crash records for these roadway sections were complied by roadway segment and time of day. The crash rates (MVM or MEV) were analyzed using formal statistical methods. The result of the analysis is a classification of the locations according to their potential nighttime hazard, which can inform decision makers of the streetlight program.

Table of Contents

List of Tables

Introduction

Staff was asked to conduct an analysis of crashes for selected City roadway segments. The City will use the results of this study as guidance for the streetlight program.

Disclaimer

The scope of work provided for this study is such that this study does not meet the requirements of a formal roadway lighting justification procedure as per Florida Department of Transportation (FDOT) Manual on Uniform Traffic Studies (MUTS). However this study in addition to others can be used as guidance with respect to crashes and countermeasures.

Procedures

The procedures in this study follow the FDOT MUTS for crash analysis, and the National Institute of Standards and Technology (NIST) e-Handbook of Engineering Data Analysis for the statistical analysis.

Data

Crash data for the selected roadway segments were obtained by querying Palm Bay Police Department traffic crash records using the *CrashStats* database tools developed by the Growth Management Department. Crash rates for a roadway segment were expressed as the number of crashes if 100 million vehicles traversed an equivalent 1-mile segment in one year. This measure is known as the MVM, and is analogous to the measure crashes per million vehicles entering (MEV), used for intersections. Details of the MVM calculations are presented in the Appendix.

Traffic volume counts on the roadway segments were conducted according to industry standards, to determine the average daily traffic on each study segment.

Analysis

The following statistical tests were conducted:

1. F-test (for unequal sample variances): The purpose of this test was to test if the underlying variability of the processes driving the crash rate overall, Citywide, was statistically the same as that of the Citywide crash rate during the day and also during the night. The F-test was also used to test if the underlying variability of the processes driving the Citywide daytime crash rates and Citywide nighttime crash rates were the same or different.

2) T-test: The T-test was used to compare the Citywide overall crash rate versus the Citywide nighttime crash rate, the Citywide overall crash rate versus the Citywide daytime crash rate, and the Citywide daytime crash rate versus the Citywide nighttime crash rate.

3) T-test: The T-test was used on each segment to determine if a) the overall crash rate on that segment was the same as or different from the overall crash rate Citywide, b) the daytime crash rate on that segment was the same as or different from the daytime Citywide crash rate, c) the nighttime crash rate on that segment was the same as or different from the nighttime Citywide crash rate, d) on a particular segment ,the daytime crash rate is the same or different from its corresponding nighttime crash rate.

Conditions a), b), and c) above can indicate whether the crash rate on a segment is above the Citywide average, and therefore would determine if the segment is a candidate for improvements/ interventions such as streetlights for nighttime crashes. Condition c) could indicate that a disproportionate number of crashes are occurring during the night regardless of whether the intersection has a higher or lower crash rate than the Citywide average.

Results

1) The results of the F-test indicate that: a) the underlying variability of the processes driving the Citywide overall crash rate and that of the Citywide daytime crash rate are fundamentally the same, b) the underlying variability of the Citywide overall crash rate and that of the Citywide nighttime crash rate are fundamentally different, and c) the underlying variability of the Citywide nighttime crash rate and that of the Citywide daytime crash rate are fundamentally different.

2) The T-tests show that the Citywide overall crash rate, the Citywide daytime crash rate, as well as the Citywide nighttime crash rate are all fundamentally the same.

3) The results of the T-test for the segments are summarized in Table 1, further details as well as the details of the F-tests are provided in the Appendix.

The T-test results for segments were used to classify the study segments into a hierarchy of concern/ priority for improvements. The classification was based on (in order of importance) a) whether the nighttime crash rate exceeded the daytime crash rate for a given segment, b) for a given segment, the nighttime crash rate exceeded the Citywide nighttime crash rate, c) the nighttime crash rate was greater than the critical crash rate for the location to be considered hazardous. The details of the calculation of the critical crash rate for each location are presented in the Appendix.

Based on these criteria the segments were classified as “most hazardous”, “hazardous” , and “not hazardous”. The results of the classification are presented in Table2.

Table 1: Summary of traffic counts and crash statistics by roadway segment

Roadway	From	To	Length (mile)	Overall			Daytime					Nighttime						MVM Hypothesis Tests			
				ADT	Crashes	100MVM	ADT	%ADT	Crashes	% Crashes	100MVM	ADT	%ADT	Crashes	% Crashes	100MVM	Critical Crash Rate (100MVM)	Overall vs Overall Ave Day	Day vs Citywide Ave Day	Night vs Citywide Ave Night	Day vs Night
Cogan	Paigo	San Filippo	0.59	2,926	1	46.40	2,295	78%	0	0%	0.00	632	22%	1	100%	214.84	542.53	less	less	less	same
Cogan	San Filippo	Babcock	1.97	2,499	31	504.44	1,870	75%	22	71%	478.41	629	25%	9	29%	581.84	542.54	same	same	same	same
Eldron	San Filippo	Babcock	1.05	2,818	14	379.03	2,127	75%	6	43%	215.22	691	25%	8	57%	883.29	542.47	same	same	greater	night > day
Hurley	Malabar	Harper	0.80	1,595[1]	5	313.91	1,252	78%	2	40%	159.96	343	22%	3	60%	875.83	543.08	same	same	greater	night > day
Malabar	Hurley	Krassner	0.50	6,347	7	176.70	4,551	72%	4	57%	140.82	1,796	28%	3	43%	267.62	541.91	same	less	less	same
Malabar	Jupiter	Hurley	0.70	10,067	40	454.72	7,726	77%	28	70%	414.75	2,341	23%	12	30%	586.63	541.80	same	same	same	same
Malabar	Garvey	Jupiter	0.54	14,155	54	565.94	10,628	75%	39	72%	544.38	3,527	25%	15	28%	630.92	541.65	greater	same	same	same
Malabar	Minton	Garvey	0.95	18,893	151	673.96	13,989	74%	123	81%	741.44	4,904	26%	28	19%	481.47[3]	541.92	greater	greater	same	same
Minton	Jupiter	Malabar	0.82	6,950	160	2249.07	5,188	75%	129	81%	2,429.16	1,762	25%	31	19%	1718.79	542.06	greater	greater	greater	day > night
San Filippo	Cogan	Eldron	0.87	4,702	3	58.75	3,376	72%	1	33%	27.27	1,326	28%	2	67%	138.88	543.91	less	less	less	same
Harper	Hurley	Garvey	1.74	705	3	195.91	530	75%	2	67%	173.73	175	25%	1	33%	263.08	542.36	less	same	less	same
San Filippo	St Andre	Cogan	1.81	3,131	6	84.81	2,321	74%	4	67%	76.28	810	26%	2	33%	109.28	542.62	less	less	less	same
Wyoming	San Filippo	Babcock	0.36	2,520	14	1236.25	1,957	78%	10	71%	1,137.07	563	22%	4	29%	1580.99	542.62	greater	greater	greater	night > day
Wyoming	Walden	San Filippo	0.68	2,520[2]	4	187.00	1,957	78%	2	50%	120.40	563	22%	2	50%	418.50	542.62	same	less	same	night > day
Lowry	San Filippo	Hemlock	0.77	544	0	0.00	410	75%	0	-	0.00	134	25%	0	-	0.00	544.32	less	less	less	same
Sarasota	Lowry	Cogan	0.88	506	1	179.91	375	74%	1	100%	242.75	131	26%	0	0%	0.00	544.36	same	same	less	same

1. One day of traffic count volumes
2. Less than one day of traffic count data was recorded, therefore the count on the abutting segment of Wyoming (from San Filippo to Babcock) was applied.
3. This location fails all statistical tests to be considered hazardous, however it marginally fails the critical crash rate test. Based on local knowledge and engineering judgment it is considered “hazardous”.

Notes:

a) Yellow highlighting indicates segments with nighttime crash rates significantly higher than daytime crash rates, as identified by the statistical hypothesis test. Also the nighttime crash rate is equal t or greater than the Citywide nighttime crash rate.
b) Blue highlighting indicates segments where nighttime crash rate is higher than Citywide nighttime crash rate, and/ or the observed crash rate is approximately equal to or greater than the critical crash rate for that segment.

Table 2: Level of concern based on nighttime crash parameters

Roadway	From	To	Level of Concern
Eldron	San Filippo	Babcock	Most hazardous
Hurley	Malabar	Harper	Most hazardous
Wyoming	San Filippo	Babcock	Most hazardous
Wyoming	Walden	San Filippo	Most hazardous
Cogan	San Filippo	Babcock	Hazardous
Malabar	Jupiter	Hurley	Hazardous
Malabar	Garvey	Jupiter	Hazardous
Malabar	Minton	Garvey	Hazardous
Minton	Jupiter	Malabar	Hazardous
Cogan	Paigo	San Filippo	Not hazardous
Malabar	Hurley	Krassner	Not hazardous
San Filippo	Cogan	Eldron	Not hazardous
Harper	Hurley	Garvey	Not hazardous
San Filippo	St Andre	Cogan	Not hazardous
Lowry	San Filippo	Hemlock	Not hazardous
Sarasota	Lowry	Cogan	Not hazardous

Conclusions

Staff was asked to conduct analyses of crashes by roadway segment, by time of day, as part of a streetlight evaluation. Crash rates were expressed per hundred million vehicles (MVM). The roadway segments were classified in a hierarchy of concern based on a statistical evaluation of the crash rates.

Appendix

Crash Query Summaries

Crash Rates

According to the Institute of Transportation Engineers (ITE) Traffic Engineering Handbook 5th Edition (1999), crash rates for roadway segments are expressed as crashes per 100 million vehicle miles of travel (100 MVM) by the equation

$$R_{SEC} = \frac{C * 100{,}000{,}000}{365 * T * V * L}$$

Where : R_{SEC} = crash rate for the roadway section/ segment

C = number of reported crashes

T = time period of the analysis (years). In this study from June 2006 to October 31, 2009, 41 months or 3.42 years

V = annual average daily traffic volume (vehicles per day). In this study Average daily traffic (ADT) was used as an estimate.

L = length of the segment (in miles)

For spots on the roadway e.g. an intersection, the crash rate is expressed as crashes per million vehicles entering the intersection (MEV) using the equation

$$R_{SPOT} = \frac{C * 1{,}000{,}000}{365 * T * V}$$

Where : R_{SPOT} = crash rate for the spot

C = number of reported crashes

T = time period of the analysis (years).

V = annual average daily traffic entering the spot (from all approaches)(vehicles per day).

The critical crash rate is calculated for each location as follows:

$$R_C = R_a + K\sqrt{\frac{R_a}{V}} + \frac{1}{2V}$$

Where : R_C = critical crash rate (100MVM or MEV, whichever is applicable)

R_a = average crash rate for locations. In this study this is the Citywide crash rate

K = constant corresponding to the level of confidence of in the conclusions of the analysis. In this study 95% confidence is adopted, therefore K = 1.645.

V = annual average daily traffic for segment, annual average daily traffic entering the spot (for spot analysis)

Statistical Analyses Presented In NIST Engineering Data Analysis e-Handbook

Reference: *NIST/SEMATECH e-Handbook of Statistical Methods*, http://www.itl.nist.gov/div898/handbook , 11/25/2009.

7.3.1. Do two processes have the same mean?

Testing hypotheses related to the means of two processes

Given two random samples of measurements,

$$Y_1, ..., Y_N \text{ and } Z_1, ..., Z_N$$

from two independent processes (the Y's are sampled from process 1 and the Z's are sampled from process 2), there are three types of questions regarding the true means of the processes that are often asked. They are:

1. Are the means from the two processes the same?
2. Is the mean of process 1 less than or equal to the mean of process 2?
3. Is the mean of process 1 greater than or equal to the mean of process 2?

Typical null hypotheses

The corresponding null hypotheses that test the true mean of the first process, μ_1, against the true mean of the second process, μ_2 are:

1. H_0: $\mu_1 = \mu_2$
2. H_0: μ_1 < or equal to μ_2
3. H_0: μ_1 > or equal to μ_2

Note that as previously discussed, our choice of which null hypothesis to use is typically made based on one of the following considerations:

1. When we are hoping to prove something new with the sample data, we make that the alternative hypothesis, whenever possible.
2. When we want to continue to assume a reasonable or traditional hypothesis still applies, unless very strong contradictory evidence is present, we make that the null hypothesis, whenever possible.

Basic statistics from the two processes

The basic statistics for the test are the sample means

$$\bar{Y} = \frac{1}{N_1}\sum_{i=1}^{N_1} Y_i \; ; \quad \bar{Z} = \frac{1}{N_2}\sum_{i=1}^{N_2} Z_i$$

and the sample standard deviations

$$s_1 = \sqrt{\frac{\sum_{i=1}^{N_1}(Y_i - \bar{Y})^2}{N_1 - 1}}$$

$$s_2 = \sqrt{\frac{\sum_{i=1}^{N_2}(Z_i - \bar{Z})^2}{N_2 - 1}}$$

with degrees of freedom $\nu_1 = N_1 - 1$ and $\nu_2 = N_2 - 1$ respectively.

Form of the test statistic where the two processes have equivalent standard deviations

If the standard deviations from the two processes are equivalent, and this should be tested before this assumption is made, the test statistic is

$$t = \frac{\bar{Y} - \bar{Z}}{s\sqrt{\frac{1}{N_1} + \frac{1}{N_2}}}$$

where the pooled standard deviation is estimated as

$$s = \sqrt{\frac{(N_1 - 1)s_1^2 + (N_2 - 1)s_2^2}{(N_1 - 1) + (N_2 - 1)}}$$

with degrees of freedom $\nu = N_1 + N_2 - 2$.

Form of the test statistic where the two processes do NOT have equivalent

If it cannot be assumed that the standard deviations from the two processes are equivalent, the test statistic is

$$t = \frac{\bar{Y} - \bar{Z}}{\sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}}}$$

standard deviations

The degrees of freedom are not known exactly but can be estimated using the Welch-Satterthwaite approximation

$$\nu = \frac{\left(\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}\right)^2}{\frac{s_1^4}{N_1^2(N_1-1)} + \frac{s_2^4}{N_2^2(N_2-1)}}$$

Test strategies

The strategy for testing the <>hypotheses under (1), (2) or (3) above is to calculate the appropriate *t* statistic from one of the formulas above, and then perform a test at significance level α, where α is chosen to be small, typically .01, .05 or .10. The hypothesis associated with each case enumerated above is rejected if:

1. $|t| \geq t_{\alpha/2;\nu}$
2. $t \geq t_{\alpha;\nu}$
3. $t \leq -t_{\alpha;\nu}$

Explanation of critical values

The critical values from the *t* table depend on the significance level and the degrees of freedom in the standard deviation. For hypothesis (1) $t_{\alpha/2;\nu}$ is the $\alpha/2$ upper critical value from the *t* table with ν degrees of freedom and similarly for hypotheses (2) and (3).

Example of unequal number of data points

A new procedure (process 2) to assemble a device is introduced and tested for possible improvement in time of assembly. The question being addressed is whether the mean, μ_2, of the new assembly process is smaller than the mean, μ_1, for the old assembly process (process 1). We choose to test hypothesis (2) in the hope that we will reject this null hypothesis and thereby feel we have a strong degree of confidence that the new process is an improvement worth implementing. Data (in minutes required to assemble a device) for both the new and old processes are listed below along with their relevant statistics.

Device	Process 1 (Old)	Process 2 (New)
1	32	36
2	37	31
3	35	30

4	28	31
5	41	34
6	44	36
7	35	29
8	31	32
9	34	31
10	38	
11	42	

Mean	36.0909	32.2222
Standard deviation	4.9082	2.5386
No. measurements	11	9
Degrees freedom	10	8

Computation of the test statistic

From this table we generate the test statistic

$$t = \frac{\bar{Y} - \bar{Z}}{\sqrt{s_1^2 / N_1 + s_2^2 / N_2}} = \frac{36.0909 - 32.2222}{\sqrt{4.9082^2 / 11 + 2.5386^2 / 9}} = 2.2694$$

with the degrees of freedom approximated by

$$\nu = \frac{\left(\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2} \right)^2}{\frac{s_1^4}{N_1^2 (N_1 - 1)} + \frac{s_2^4}{N_2^2 (N_2 - 1)}} = \frac{\left(\frac{4.9082^2}{11} + \frac{2.5386^2}{9} \right)^2}{\frac{4.9082^4}{1210} + \frac{2.5386^4}{648}} = 15.5$$

Decision process

For a one-sided test at the 5% significance level, go to the *t* table for 5% signficance level, and look up the critical value for degrees of freedom ν= 16. The critical value is 1.746. Thus, hypothesis (2) is rejected because the test statistic (t = 2.269) is greater than 1.746 and, therefore, we conclude that process 2 has improved assembly time (smaller mean) over process 1.

7.3.2. Do two processes have the same standard deviation?

Testing hypotheses related to standard deviations from two processes

Given two random samples of measurements,

$$\boldsymbol{Y_1}, \ldots, \boldsymbol{Y_N} \quad \text{and} \quad \boldsymbol{Z_1}, \ldots, \boldsymbol{Z_N}$$

from two independent processes, there are three types of questions regarding the true standard deviations of the processes that can be addressed with the sample data. They are:

1. Are the standard deviations from the two processes the same?
2. Is the standard deviation of one process less than the standard deviation of the other process?
3. Is the standard deviation of one process greater than the standard deviation of the other process?

Typical null hypotheses

The corresponding null hypotheses that test the true standard deviation of the first process, σ_1, against the true standard deviation of the second process, σ_2 are:

1. H_0: $\sigma_1 = \sigma_2$
2. H_0: $\sigma_1 \le \sigma_2$
3. H_0: $\sigma_1 \ge \sigma_2$

Basic statistics from the two processes

The basic statistics for the test are the sample variances

$$s_1^2 = \frac{1}{N_1 - 1} \sum_{i=1}^{N1} (Y_i - \bar{Y})^2$$

$$s_2^2 = \frac{1}{N_2 - 1} \sum_{i=1}^{N2} (Z_i - \bar{Z})^2$$

and degrees of freedom $\nu_1 = N_1 - 1$ and $\nu_2 = N_2 - 1$, respectively.

Form of the test statistic

The test statistic is

$$F = \frac{s_1^2}{s_2^2}$$

Test strategies

The strategy for testing the <>hypotheses under (1), (2) or (3) above is to calculate the F statistic from the formula above, and then perform a test at significance level α, where α is chosen to be small, typically .01, .05 or .10. The hypothesis associated with each case enumerated above is rejected if:

1. $F \le \frac{1}{F_{\alpha/2;\nu 2;\nu 1}}$ or $F \ge F_{\alpha/2;\nu 1;\nu 2}$
2. $F \ge F_{\alpha;\nu 1;\nu 2}$
3. $F \le \frac{1}{F_{\alpha;\nu 2;\nu 1}}$

Explanation of critical values

The critical values from the *F* table depend on the significance level and the degrees of freedom in the standard deviations from the two processes. For hypothesis (1):

- $F_{\alpha/2;\, \nu_2;\, \nu_1}$ is the upper critical value from the F table with
- $\nu_2 = N_2 - 1$ degrees of freedom for the numerator and
- $\nu_1 = N_1 - 1$ degrees of freedom for the denominator

and

- $F_{\alpha/2;\, \nu_1;\, \nu_2}$ is the upper critical value from the F table with
- $\nu_1 = N_1 - 1$ degrees of freedom for the numerator and
- $\nu_2 = N_2 - 1$ degrees of freedom for the denominator.

***Caution** on looking up critical values*

The F distribution has the property that

$$F_{1-\alpha/2;\, \nu_1;\, \nu_2} = \frac{1}{F_{\alpha/2;\, \nu_2;\, \nu_1}}$$

which means that only upper critical values are required for two-sided tests. However, note that the degrees of freedom are interchanged in the ratio. For example, for a two-sided test at significance level 0.05, go to the *F* table labeled "2.5% significance level".

- For $F_{\alpha/2;\, \nu_2;\, \nu_1}$, reverse the order of the degrees of freedom; i.e., look across the top of the table for $\nu_2 = N_2 - 1$ and down the table for $\nu_1 = N_1 - 1$.

- For $F_{\alpha/2;\,\nu_1;\,\nu_2}$, look across the top of the table for $\nu_1 = N_1 - 1$ and down the table for $\nu_2 = N_2 - 1$.

Critical values for cases (2) and (3) are defined similarly, except that the critical values for the one-sided tests are based on α rather than on $\alpha/2$.

Two-sided confidence interval

The two-sided confidence interval for the ratio of the two unknown variances (squares of the standard deviations) is shown below.

Two-sided confidence interval with 100(1-α)% coverage for:

$$\frac{\sigma_1^2}{\sigma_2^2} \qquad \frac{1}{F_{\alpha/2;N1-1;N2-1}}\left(\frac{s_1^2}{s_2^2}\right),\ F_{\alpha/2;N2-1;N1-1}\left(\frac{s_1^2}{s_2^2}\right)$$

One interpretation of the confidence interval is that if the quantity "one" is contained within the interval, the standard deviations are equivalent.

Example of unequal number of data points

A new procedure to assemble a device is introduced and tested for possible improvement in time of assembly. The question being addressed is whether the standard deviation, σ_2, of the new assembly process is better (i.e., smaller) than the standard deviation, σ_1, for the old assembly process. Therefore, we test the null hypothesis that $\sigma_1 \le \sigma_2$. We form the hypothesis in this way because we hope to reject it, and therefore accept the alternative that σ_2 is less than σ_1. This is hypothesis (2). Data (in minutes required to assemble a device) for both the old and new processes are listed on an earlier page. Relevant statistics are shown below:

	Process 1	Process 2
Mean	36.0909	32.2222
Standard deviation	4.9082	2.5874
No. measurements	11	9
Degrees freedom	10	8

Computation of the test statistic

From this table we generate the test statistic

$$F = \frac{s_1^2}{s_2^2} = \left(\frac{4.9082}{2.5874}\right)^2 = 3.60$$

Decision process

For a test at the 5% significance level, go to the [*F* table for 5% signficance level](), and look up the critical value for numerator degrees of freedom $\nu_1 = N_1 - 1$ = 10 and denominator degrees of freedom $\nu_2 = N_2 - 1$ = 8. The critical value is 3.35. Thus, hypothesis (2) can be rejected because the test statistic (F = 3.60) is greater than 3.35. Therefore, we accept the alternative hypothesis that process 2 has better precision (smaller standard deviation) than process 1.

7.2.2. Are the data consistent with the assumed process mean?

The testing of H_0 for a single population mean

Given a random sample of measurements, Y_1, ..., Y_N, there are three types of questions regarding the true mean of the population that can be addressed with the sample data. They are:

1. Does the true mean agree with a known standard or assumed mean?
2. Is the true mean of the population less than a given standard?
3. Is the true mean of the population at least as large as a given standard?

Typical null hypotheses

The corresponding null hypotheses that test the true mean, μ, against the standard or assumed mean, μ_0 are:

1. $H_0: \mu = \mu_0$
2. $H_0: \mu \le \mu_0$
3. $H_0: \mu \ge \mu_0$

Test statistic

The basic statistics for the test are the sample mean and the *where the standard deviation is not known*

standard deviation. The form of the test statistic depends on whether the poulation standard deviation, σ, is known or is estimated from the data at hand. The more typical case is where the standard deviation must be estimated from the data, and the test statistic is

$$t = \frac{\bar{Y} - \mu_0}{s / \sqrt{N}}$$

where the sample mean is

$$\bar{Y} = \frac{1}{N} \sum_{i=1}^{N} Y_i$$

and the sample standard deviation is

$$s = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} \left(Y_i - \bar{Y} \right)^2}$$

with N - 1 degrees of freedom.

Comparison with critical values

For a test at significance level α, where α is chosen to be small, typically .01, .05 or .10, the hypothesis associated with each case enumerated above is rejected if:

1. $|t| \ge t_{\alpha/2;\, N-1}$
2. $t \ge t_{\alpha;\, N-1}$
3. $t \le -t_{\alpha;\, N-1}$

where $t_{\alpha/2;\, N-1}$ is the upper $\alpha/2$ critical value from the *t* distribution with *N-1* degrees of freedom and similarly for cases (2) and (3). Critical values can be found in the t-table in Chapter 1.

Test statistic where the

If the standard deviation is known, the form of the test statistic is *standard deviation is known*

$$z = \frac{\bar{Y} - \mu_0}{\sigma / \sqrt{N}}$$

For case (1), the test statistic is compared with $z_{\alpha/2}$, which is the upper $\alpha/2$ critical value from the standard normal distribution, and similarly for cases (2) and (3).

Caution

If the standard deviation is assumed known for the purpose of this test, this assumption should be checked by a test of hypothesis for the standard deviation.

An illustrative example of the t-test

The following numbers are particle (contamination) counts for a sample of 10 semiconductor silicon wafers:

```
50   48   44   56   61   52   53   55   67   51
```

The mean = 53.7 counts and the standard deviation = 6.567 counts.

The test is two-sided

Over a long run the process average for wafer particle counts has been 50 counts per wafer, and on the basis of the sample, we want to test whether a change has occurred. The null hypothesis that the process mean is 50 counts is tested against the alternative hypothesis that the process mean is not equal to 50 counts. The purpose of the two-sided alternative is to rule out a possible process change in either direction.

Critical values

For a significance level of α = .05, the chances of erroneously rejecting the null hypothesis when it is true are 5% or less. (For a review of hypothesis testing basics, see Chapter 1).

Even though there is a history on this process, it has not been stable enough to justify the assumption that the standard deviation is known. Therefore, the appropriate test statistic is the *t*-statistic. Substituting the sample mean, sample standard deviation, and sample size into the formula for the test statistic gives a value of

$$t = 1.782$$

with degrees of freedom = N - 1 = 9. This value is tested against the upper critical value

$$t_{0.025;9} = 2.262$$

from the t-table where the critical value is found under the column labeled 0.025 for the probability of exceeding the critical value and in the row for 9 degrees of freedom. The critical value $\alpha/2$ is used instead of α because of the two-sided alternative (two-tailed test) which requires equal probabilities in each tail of the distribution that add to α.

Conclusion

Because the value of the test statistic falls in the interval (-2.262, 2.262), we cannot reject the null hypothesis and, therefore, we may continue to assume the process mean is 50 counts.

Analysis of the Data

Outliers were removed from each dataset to prevent skewing of the results.

Overall crash rates per segment

	46.40392
	504.4408
	379.0345
	313.9065
	176.7016
	454.7188
	565.941
	673.959
	58.74899
	195.9133
	84.8146
	1236.249
	186.9957
	179.9069
mean	361.2668
pop SD	308.0819
sample SD	319.7117

Daytime crash rates per segment

478.40518
215.21646
159.96196
140.81995
414.74993
544.37771
741.43975
27.27466
173.73444
76.275894
1137.0713
120.39579
242.75439

mean	344.03672
pop SD	304.2153
sample SD	316.63732

Nighttime crash rates per segment

214.84
581.84
883.29
875.83
267.62
586.63
630.92
481.47
138.88
263.08
109.28
1580.99
418.50

mean	541.01
pop SD	387.0676
sample SD	402.8727

1.Citywide Overall Crash rate versus Citywide Daytime Crash rate

F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	361.2668	344.0367
Variance	102215.6	100259.2
Observations	14	13
df	13	12
F	1.019513	
P(F<=f) one-tail	0.489698	
F Critical one-tail	2.660177	

Conclusion: Overall SD and daytime SD are the same t-Test: Two-Sample Assuming Equal Variances

	Variable 1	*Variable 2*
Mean	361.2668	344.0367
Variance	102215.6	100259.2
Observations	14	13
Pooled Variance	101276.5	
Hypothesized Mean Difference	0	
df	25	
t Stat	0.140568	
P(T<=t) one-tail	0.444669	
t Critical one-tail	1.708141	
P(T<=t) two-tail	0.889338	
t Critical two-tail	2.059539	

Conclusion: Overall crash rate and daytime crash rate are the same

<u>2.Citywide Overall Crash rate versus Citywide Nightime Crash rate</u>

F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	361.2668	541.0135
Variance	102215.6	162306.4
Observations	14	13
df	13	12
F	0.629769	
P(F<=f) one-tail	0.209777	
F Critical one-tail	0.384075	

Conclusion: Nighttime SD different than overall t-Test: Two-Sample Assuming Unequal Variances

	Variable 1	*Variable 2*
Mean	361.2668	541.0135
Variance	102215.6	162306.4
Observations	14	13
Hypothesized Mean Difference	0	
df	23	
t Stat	-1.27785	
P(T<=t) one-tail	0.107025	
t Critical one-tail	1.713872	
P(T<=t) two-tail	0.21405	
t Critical two-tail	2.068658	

Conclusion: Overall crash rate and nighttime crash rate are the same

<u>3.Citywide Daytime Crash rate versus Citywide Nighttime Crash rate</u>

F-Test Two-Sample for Variances

	Variable 1	*Variable 2*
Mean	344.036724	541.0135431
Variance	100259.1941	162306.4317
Observations	13	13
df	12	12
F	0.617715472	
P(F<=f) one-tail	0.208000981	
F Critical one-tail	0.372212531	

Conclusion: reject null hypothesis. daytime and nighttime SDs are different t-Test: Two-Sample Assuming Unequal Variances

	Variable 1	*Variable 2*
Mean	344.036724	541.0135431
Variance	100259.1941	162306.4317
Observations	13	13
Hypothesized Mean Difference	0	
df	23	
t Stat	-1.386014781	
P(T<=t) one-tail	0.089517274	
t Critical one-tail	1.713871517	
P(T<=t) two-tail	0.179034547	
t Critical two-tail	2.068657599	

Conclusion: Overall crash rate and nighttime crash rate are statistically the same

General Conclusion:

There are differences in variance, but the underlying process for overall, daytime, and nighttime are the same according to the t-tests.

We can use Citywide “overall”, Citywide “daytime”, or Citywide “nighttime” averages for the comparison to segments since the t-tests show they are all statistically the same. In this analysis we used nighttime to compare segment nighttime etc . This approach will yield the same result as we have shown Citywide nighttime, Citywide daytime, Citywide overall are all the same.

Hypothesis Tests per Segment - *Minitab* Output

————— 11/24/2009 4:14:12 PM ————————————————————

Welcome to Minitab, press F1 for help.

One-Sample T

Test of mu = 46.4039 vs < 46.4039

N	Mean	StDev	SE Mean	95% Upper Bound	T	P
13	361.3	308.1	85.4	513.6	3.68	0.998

One-Sample T

Test of mu = 46.4039 vs > 46.4039

N	Mean	StDev	SE Mean	95% Lower Bound	T	P
13	361.3	308.1	85.4	209.0	3.68	0.002

One-Sample T

Test of mu = 46.4039 vs not = 46.4039

N	Mean	StDev	SE Mean	95% CI	T	P
13	361.3	308.1	85.4	(175.1, 547.4)	3.68	0.003

<u>Confidence Interval for Citywide overall crash rate</u>

One-Sample T

Test of mu = 504.44 vs > 504.44

N	Mean	StDev	SE Mean	95% Lower Bound	T	P
13	361.3	308.1	85.4	209.0	-1.68	0.940

One-Sample T

Test of mu = 504.44 vs < 504.44

				95% Upper		
N	Mean	StDev	SE Mean	Bound	T	P
13	361.3	308.1	85.4	513.6	-1.68	0.060

One-Sample T

Test of mu = 504.44 vs not = 504.44

N	Mean	StDev	SE Mean	95% CI	T	P
13	361.3	308.1	85.4	(175.1, 547.4)	-1.68	0.120

One-Sample T

Test of mu = 478.41 vs not = 478.41

N	Mean	StDev	SE Mean	95% CI	T	P	
12	344.0	316.6	91.4	(142.9, 545.2)	-1.47	0.170	**Confidence Interval for City daytime crash rate**

One-Sample T

Test of mu = 478.41 vs not = 478.41

N	Mean	StDev	SE Mean	95% CI	T	P	
12	541	387	112	(295, 787)	0.56	0.587	**Confidence Interval for Citywide nighttime crash rate**

Two-Sample T-Test and CI

Sample	N	Mean	StDev	SE Mean
1	12	0	304	88
2	12	215	387	112

Difference = mu (1) - mu (2)

Estimate for difference: -215

95% CI for difference: (-511, 82)

T-Test of difference = 0 (vs not =): T-Value = -1.51 P-Value = 0.146 DF = 20

Two-Sample T-Test and CI

Sample	N	Mean	StDev	SE Mean
1	12	478	304	88

2	12	582	387	112

Difference = mu (1) - mu (2)

Estimate for difference: -103

95% CI for difference: (-400, 193)

T-Test of difference = 0 (vs not =): T-Value = -0.73 P-Value = 0.475 DF = 20

Two-Sample T-Test and CI

Sample	N	Mean	StDev	SE Mean
1	12	215	304	88
2	12	883	387	112

Difference = mu (1) - mu (2)

Estimate for difference: -668

95% CI for difference: (-965, -372)

T-Test of difference = 0 (vs not =): T-Value = -4.70 P-Value = 0.000 DF = 20

Two-Sample T-Test and CI

Sample	N	Mean	StDev	SE Mean

1	12	160	304	88
2	12	876	387	112

Difference = mu (1) - mu (2)

Estimate for difference: -716

95% CI for difference: (-1012, -419)

T-Test of difference = 0 (vs not =): T-Value = -5.04 P-Value = 0.000 DF = 20

Two-Sample T-Test and CI

Sample	N	Mean	StDev	SE Mean
1	12	141	304	88
2	12	268	387	112

Difference = mu (1) - mu (2)

Estimate for difference: -127

95% CI for difference: (-423, 170)

T-Test of difference = 0 (vs not =): T-Value = -0.89 P-Value = 0.383 DF = 20

Two-Sample T-Test and CI

Sample	N	Mean	StDev	SE Mean
1	12	415	304	88
2	12	587	387	112

Difference = mu (1) - mu (2)

Estimate for difference: -172

95% CI for difference: (-468, 125)

T-Test of difference = 0 (vs not =): T-Value = -1.21 P-Value = 0.241 DF = 20

Two-Sample T-Test and CI

Sample	N	Mean	StDev	SE Mean
1	12	544	304	88
2	12	631	387	112

Difference = mu (1) - mu (2)

Estimate for difference: -87

95% CI for difference: (-383, 210)

T-Test of difference = 0 (vs not =): T-Value = -0.61 P-Value = 0.549 DF = 20

Two-Sample T-Test and CI

Sample	N	Mean	StDev	SE Mean
1	12	741	304	88
2	12	481	387	112

Difference = mu (1) - mu (2)

Estimate for difference: 260

95% CI for difference: (-36, 556)

T-Test of difference = 0 (vs not =): T-Value = 1.83 P-Value = 0.082 DF = 20

Two-Sample T-Test and CI

Sample	N	Mean	StDev	SE Mean
1	12	2429	304	88
2	12	1719	387	112

Difference = mu (1) - mu (2)

Estimate for difference: 710

95% CI for difference: (414, 1007)

T-Test of difference = 0 (vs not =): T-Value = 5.00 P-Value = 0.000 DF = 20

Two-Sample T-Test and CI

Sample	N	Mean	StDev	SE Mean
1	12	27	304	88
2	12	139	387	112

Difference = mu (1) - mu (2)

Estimate for difference: -112

95% CI for difference: (-408, 185)

T-Test of difference = 0 (vs not =): T-Value = -0.79 P-Value = 0.441 DF = 20

Two-Sample T-Test and CI

Sample	N	Mean	StDev	SE Mean
1	12	174	304	88
2	12	263	387	112

Difference = mu (1) - mu (2)

Estimate for difference: -89

95% CI for difference: (-386, 207)

T-Test of difference = 0 (vs not =): T-Value = -0.63 P-Value = 0.537 DF = 20

Two-Sample T-Test and CI

Sample	N	Mean	StDev	SE Mean
1	12	76	304	88
2	12	109	387	112

Difference = mu (1) - mu (2)

Estimate for difference: -33

95% CI for difference: (-329, 263)

T-Test of difference = 0 (vs not =): T-Value = -0.23 P-Value = 0.819 DF = 20

Two-Sample T-Test and CI

Sample	N	Mean	StDev	SE Mean
1	12	1137	304	88
2	12	1581	387	112

Difference = mu (1) - mu (2)

Estimate for difference: -444

95% CI for difference: (-740, -147)

T-Test of difference = 0 (vs not =): T-Value = -3.12 P-Value = 0.005 DF = 20

Two-Sample T-Test and CI

Sample	N	Mean	StDev	SE Mean
1	12	120	304	88
2	12	419	387	112

Difference = mu (1) - mu (2)

Estimate for difference: -298

95% CI for difference: (-595, -2)

T-Test of difference = 0 (vs not =): T-Value = -2.10 P-Value = 0.049 DF = 20

Two-Sample T-Test and CI

Sample	N	Mean	StDev	SE Mean
1	12	0	304	88
2	12	0	387	112

Difference = mu (1) - mu (2)

Estimate for difference: 0

95% CI for difference: (-296, 296)

T-Test of difference = 0 (vs not =): T-Value = 0.00 P-Value = 1.000 DF = 20

Two-Sample T-Test and CI

Sample	N	Mean	StDev	SE Mean
1	12	243	304	88
2	12	0	387	112

Difference = mu (1) - mu (2)

Estimate for difference: 243

95% CI for difference: (-54, 539)

T-Test of difference = 0 (vs not =): T-Value = 1.71 P-Value = 0.103 DF = 20