The data below resulted from measuring the performance of a bridge component subjected to three different environments. The engineer wants to test the claim that the component performs the same in all environments. The test shall be conducted at 95% confidence level.

Environment (Treatment)	Mild (1)	Moderate (2)	Aggressive (3)
	6.9	8.3	8.0
	5.4	6.8	10.5
	5.8	7.8	8.1
	4.6	9.2	6.9
	4.0	6.5	9.3
			7.4

H_0: $\mu_1 = \mu_2 = \mu_3$
H_a: not all the same, at least on different

$\alpha = 0.05$

So we have $k = 3$ treatments (samples), $N = 16$ measurements overall.

First of all let's compile the descriptive statistics for each treatment (sample)

Treatment 1:

Sample size $n_1 = 5$, degrees of freedom, $df_1 = 5 - 1 = 4$

Mean,

$$\bar{x}_1 = \frac{6.9 + 5.4 + 5.8 + 4.6 + 4.0}{5} = 5.34$$

Sum of Square Errors (SSE) for Treatment 1,

$$SSE_1 = \sum_{i=1}^{n_1=5} (x_i - \bar{x}_1)^2$$

$$SSE_1 = (6.9 - 5.34\quad)^2 + (5.4 - 5.34\quad)^2 + (5.8 - 5.34\quad)^2 + (4.6 - 5.34\quad)^2$$
$$+(4.0 - 5.34\quad)^2 = 4.992$$

So a value minus the sample (treatment) mean is called an Error. We square the errors and sum them up, hence the name Sum of Square Errors. SSE measures the amount of variation within the treatment.

Treatment 2:

Sample size n_2 = 5, degrees of freedom, df_2 = 5 – 1 = 4

Mean,

$$\bar{x}_2 = \frac{8.3 + 6.8 + 7.8 + 9.5 + 6.5}{5} = 7.72$$

Sum of Square Errors (SSE) for Treatment 1,

$$SSE_2 = \sum_{i=1}^{n_2=5} (x_i - \bar{x}_2)^2$$

$$SSE_2 = (8.3 - 7.72\quad)^2 + (6.8 - 7.72\quad)^2 + (7.8 - 7.72\quad)^2 + (9.5 - 7.72\quad)^2$$
$$+(6.5 - 7.72\quad)^2 = 4.868$$

Treatment 3:

Sample size n_3 = 6, degrees of freedom, df_3 = 6 – 1 = 5

Mean,

$$\bar{x}_3 = \frac{8.0 + 10.5 + 8.1 + 6.9 + 9.3 + 7.4}{6} = 8.37$$

Sum of Square Errors (SSE) for Treatment 3,

$$SSE_3 = \sum_{i=1}^{n_3=6} (x_i - \bar{x}_3)^2$$

$$SSE_3 = (8.0 - 8.37)^2 + (10.5 - 8.37)^2 + (8.1 - 8.37)^2 + (6.9 - 8.37)^2 + (9.3 - 8.37)^2$$
$$+(7.4 - 8.37)^2 = 8.7136$$

Overall

The overall mean is

$$\bar{x} = \frac{6.9 + 5.4 + 5.8 + 4.6 + 4.0 + 8.3 + 6.8 + 7.8 + 9.5 + 6.5 + 8.0 + 10.5 + 8.1 + 6.9 + 9.3 + 7.4}{16}$$

$$\bar{x} = 7.243$$

The overall degrees of freedom for the errors is the sum of the values for each treatment

$$df_{total} = df_1 + df_2 + df_3 = (n_1 - 1) + (n_2 - 1) + (n_3 - 1) = N - k = 16 - 3 = 13$$

The overall Sum of Squares for Errors is the sum of the Sum of Squares for Errors for each treatment

$$SSE = SSE_1 + SSE_2 + SSE_3 = 4.992 + 4.868 + 8.7136 = 18.5733$$

Another measure of variation in the data is the Sum of Squares for Treatments (SST). This measures the variation across or between samples (treatments). It is calculated as follows,

$$SST = \sum_{i=1}^{k=3} n_i(\bar{x}_i - \bar{x})^2$$

In other words sample size times the square of sample mean minus the overall mean. Repeat for all samples (treatments) and sum them up.

$$SST = 5(5.34 - 7.243)^2 + 5(7.72 - 7.243)^2 + 6(8.37 - 7.243)^2 = 26.81104$$

Degrees of freedom for treatments is $(k - 1) = 3 - 1 = 2$

We may now update our summary table as follows,

Environment (Treatment)	Mild (1)	Moderate (2)	Aggressive (3)	Overall
	6.9	8.3	8.0	
	5.4	6.8	10.5	
	5.8	7.8	8.1	
	4.6	9.2	6.9	
	4.0	6.5	9.3	
			7.4	
Means	5.34	7.72	8.37	7.243
df	4	4	5	
SSE	4.992	4.868	8.7136	
SST				26.8110

We may now compile the ANOVA table. The Mean Squares are calculated by dividing the Sum of Squares by their corresponding degrees of freedom.

Sources of variation	**Degrees of freedom**	**Sum of Squares**	**Mean Squares**	**F**	**p-value**
Treatments	$k - 1$	SST	$MST = SST/(k-1)$	$F_{calc}=MST/MSE$	α_{calc} for F_{calc}
Errors	$N - k$	SSE	$MSE = SSE/(N-k)$		
Totals	$N - 1$	$SS(total)$			

For our case the ANOVA table is as follows,

Sources of variation	**Degrees of freedom**	**Sum of Squares**	**Mean Squares**	**F**	
Treatments	*2*	*26.811*	*13.4055*	$F_{calc}=9.3829$	*0.003005*
Errors	*13*	*18.5733*	*1.4287*		
Totals	*15*	*45.3843*			

Now the critical value is as follows,

$$F_{critical} = F_{\alpha=0.05,\ k-1=2, N-k=13} = 3.8055$$

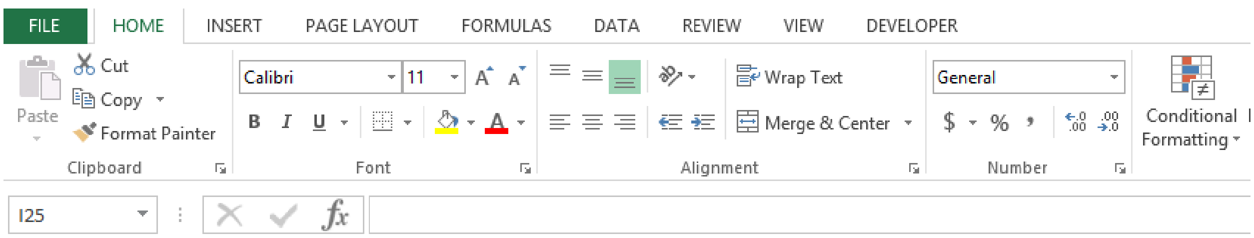
So our $F_{calc} > F_{critical}$

Or if you prefer p-values, our $\alpha = 0.05 > p - value = 0.003005$

So we are clearly in the rejection region. REJECT H_0. So bridge component does not perform the same in all environments. At least one is different.

Students, please see ANOVA part 2 and part 3 videos to see how the different one(s) can be identified.

Excel has ANOVA button. My data and output is follows:



	A	B	C	D	E	F	G	H	I	J	K	L
1	6.9	8.3	8		Anova: Single Factor							
2	5.4	6.8	10.5									
3	5.8	7.8	8.1		SUMMARY							
4	4.6	9.2	6.9		*Groups*	*Count*	*Sum*	*Average*	*Variance*			
5	4	6.5	9.3		Column 1	5	26.7	5.34	1.248			
6			7.4		Column 2	5	38.6	7.72	1.217			
7					Column 3	6	50.2	8.366667	1.742667			
8												
9												
10					ANOVA							
11					*Source of Variation*	*SS*	*df*	*MS*	*F*	*P-value*	*F crit*	
12					Between Groups	26.81104	2	13.40552	9.382902	0.003005	3.805565	
13					Within Groups	18.57333	13	1.428718				
14												
15					Total	45.38438	15					
16												
17												