P.34

p.162

X = substrate concentration in mg/cm^3

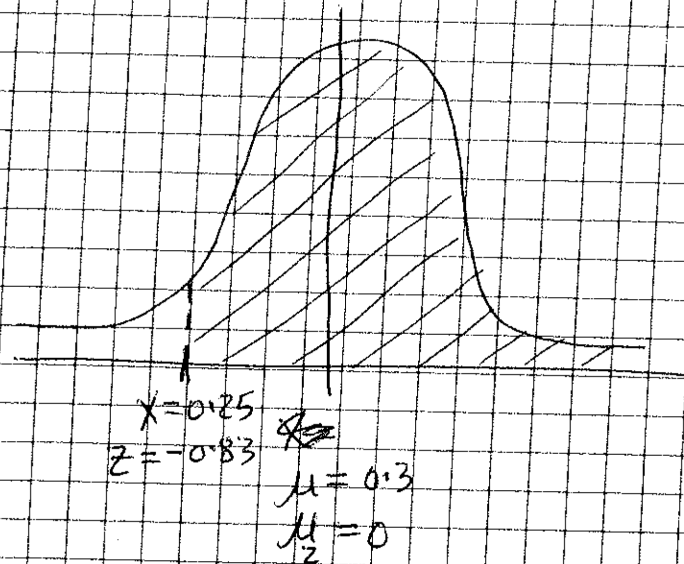
$X \sim \text{Normal}(\mu = 0.3, \sigma = 0.06)$

a) $P(X > 0.25) = ?$

so first we must convert our random variable to a Standard Normal distribution using the transformation

$$Z = \frac{X - \mu}{\sigma}$$

$$Z = \frac{0.25 - 0.3}{0.06} = -0.83$$



So we go to the Standard Normal table to look up the area under the curve.

Note that the area shaded under the curve on the table is to the left. We want the area to the right, so must do complement, 1 minus the value we get from the table.

So

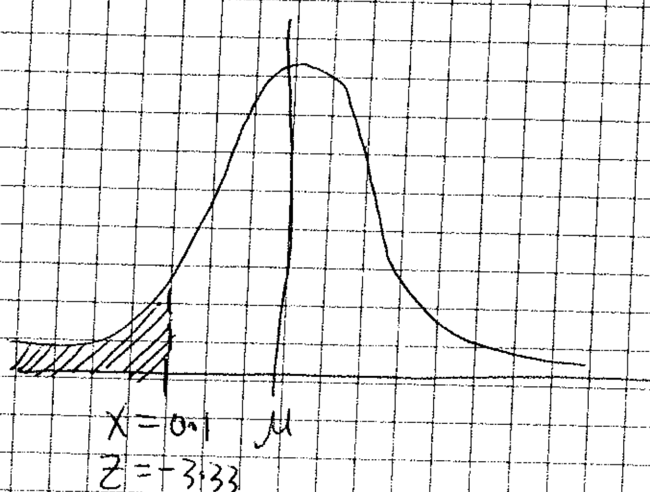
$$P(X > 0.25) = P(Z > -0.83)$$

$$= 1 - 0.2033$$

$$= 0.7967$$

b) $P(X \leq 0.1) = ?$

$$Z = \frac{X - \mu}{\sigma} = \frac{0.1 - 0.3}{0.06} = -3.33$$

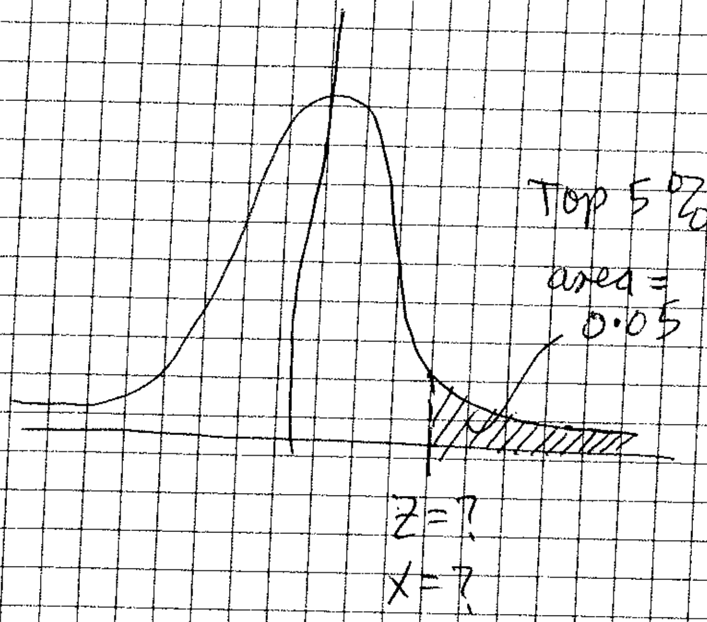


$$P(X \leq 0.1) = P(Z < -3.33) \xrightarrow{\text{table}}$$

$$= 0.0004$$

Note that the area we were looking for was to the left of the Z-score, which is how the area is shaded on the table, so we did not need to subtract our table pred value from 1 as we did in P part (a).

c) So here we have been given a probability value and need to work backwards to find the Z-score and then the real X value.



Our table shows areas to the left. So we must subract to given area from 1 to get the area to the left.

$$1 - 0.05 = 0.95$$

Now go to the table and look at the area (probability) values and find 0.95, or the closest value to it.

Can we see that

area $= 0.95 \longrightarrow z = 1.645$

So if $z = 1.645$

the $1.645 = \dfrac{X - 0.3}{0.06}$

$$X = 0.3 + 1.645(0.06)$$

$$= 0.3987 \ gm/cm^3$$

So any value greater than 0.3987 falls in the top 5%. Or we can say the 95^{th} percentile is 0.3987 gm/cm³.

P.36

Page 162

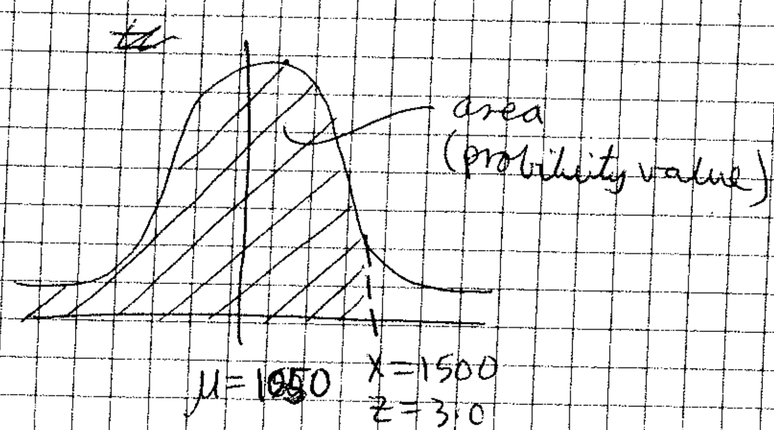
X = size of spray droplets

$X \sim \text{Normal}(\mu = 1050\ \mu m, \sigma = 150\ \mu m)$

a) $P(X < 1500) = ?$

$$Z = \frac{1500 - 1050}{150} = 3.0$$

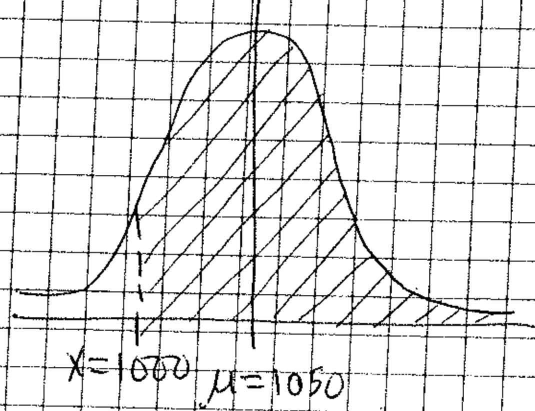
$P(X < 1500) = P(Z < 3.0) \rightarrow$ table



$= 0.9987$

$P(X \geqslant 1000)$

$$Z = \frac{1000 - 1050}{150} = -0.33$$



$P(X \geqslant 1000) = P(Z \geqslant -0.33) \rightarrow$ table

$= 1 - 0.3707$

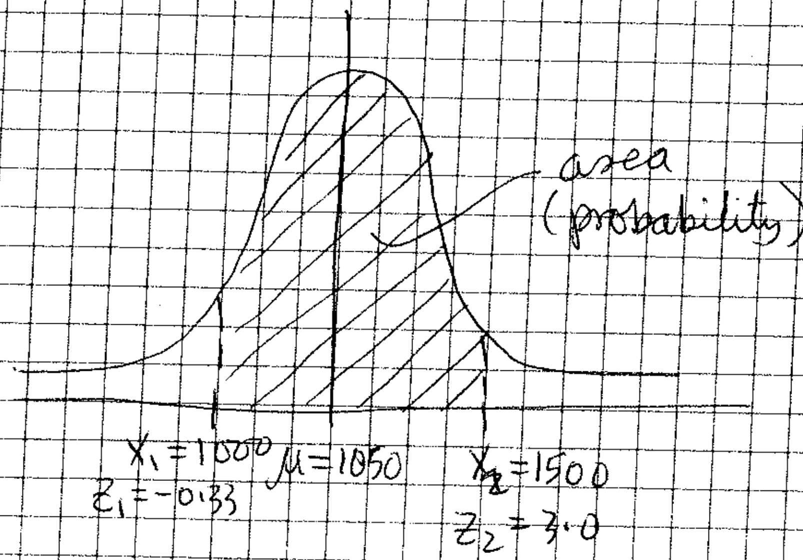
$= 0.6293$

b) $P(1000 < X < 1500)$

so now we have 2 Zs.

$$Z_1 = \frac{1000 - 1050}{150} = -0.33$$

$$Z_2 = \frac{1500 - 1050}{150} = 3.0$$



$$P(1000 < X < 1500) = P(-0.33 < Z < 3.0)$$

$$\cancel{= P(Z < 3.0) - P(Z < -0.33)}$$

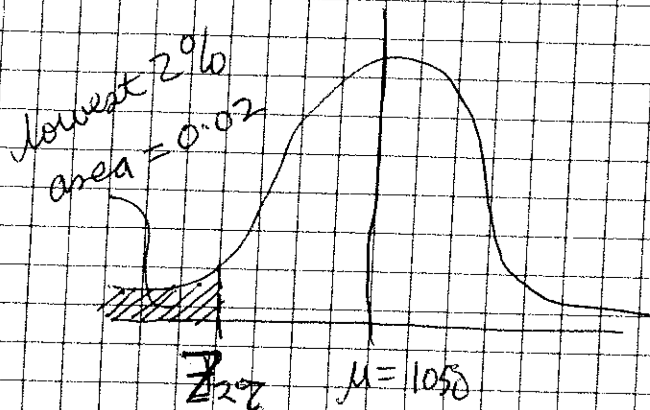
$$= P(Z_2 < 3.0) - P(Z_1 < -0.33)$$

– go to tables –

$$= 0.9987 - \cancel{0.6293}\ 0.3707$$

$$= \cancel{0.3694}\ 0.628$$

c) The lowest 2%. In other words what is the 2nd percentile, P_2?



From table,

area = 0.02 $\longrightarrow$ Z = ?

area :	0.0202	0.02	0.0197
	$\downarrow$	$\downarrow$	$\downarrow$
z :	-2.05	$Z_{2\%}$	-2.06

$$\frac{Z_{2\%} - (-2.05)}{0.02 - 0.0202} = \frac{-2.06 - (-2.05)}{0.0197 - 0.0202}$$

$$\Rightarrow Z_{2\%} = -2.054$$

Now this is a z-score

so

~~$X_{2\%}$ =~~

$$z = \frac{x-\mu}{\sigma} \Rightarrow \frac{X_{2\%}-1050}{150} = -2.054$$

so $X_{2\%} = \cancel{-2.54\left(\frac{150}{0.06}\right)+1050}$
$= 741.9\ \mu m$

~~= 4049.85 μm~~

2nd Percentile ~~= 1669 μm~~
droplet
size
$= 741.9\ \mu m$

d) $n = 5$

$$P(X > 1500) = 1 - P(X < 1500)$$

we already Calculated this, otherwise go through Normal Table process.

$$P(X > 1500) = 1 - 0.9987$$
$$= 0.0013$$

Of these 5 droplets we want find probability that at least one exceeds 1500 μm.

So we pick one droplet. Exceed or not exceed 1500 μm. What are we doing? Binomial Theorem?!?!

$$P(X \geq 1) = P(X=1) + P(X=2) + P(X=3) + P(X=4) \; \cancel{\text{or}} + P(X=5) \text{ or}$$

$$= 1 - P(X<1)$$

$$= 1 - P(X=0)$$

$$= 1 - {}^{5}C_{0} (0.0013)^{0} (1-0.0013)^{5-0}$$

$$=$$