Sampling Distributions

A City traffic engineer is monitoring vehicle speeds on a county roadway. Due to time and other limitations the engineer cannot monitor all the vehicles that drive this roadway. So the engineer takes a random sample of 60 vehicles. From the sample the engineer found that the average speed is 48 mph, with a standard deviation of 3.2 mph.

1) What is the probability that a vehicle selected at random will driving less than 46.5 mph?

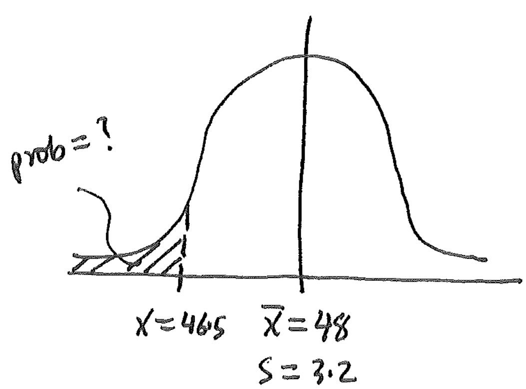
2) What is the 85th percentile speed on this roadway?

Now the County is conducting a study of its own. However the County engineer selected a random sample of 25 vehicles yielding an average speed of 46.2 mph with a standard deviation of 4.3 mph.

3) What is the probability that a vehicle selected at random will be traveling at less than 45.5 mph?

4) According to the County engineer's sample what is the 85th percentile speed on this roadway?

1. $n = 60 > 30$ so use Normal distribution



General formula:

$$P(X \leq x) = P\left(Z \leq \frac{x - \bar{X}}{s/\sqrt{n}}\right)$$

$$P(X \leq 46.5) = P\left(Z \leq \frac{46.5 - 48}{3.2/\sqrt{60}}\right) = P(Z \leq -3.63)$$

So we go to table and look up $Z = -3.63$. If we cannot find it we have to interpolate or select closest value.

From Table

z: -3.0 $\quad\downarrow\quad$ p: 0.0013

z: -3.4 $\quad\downarrow\quad$ p: 0.0003

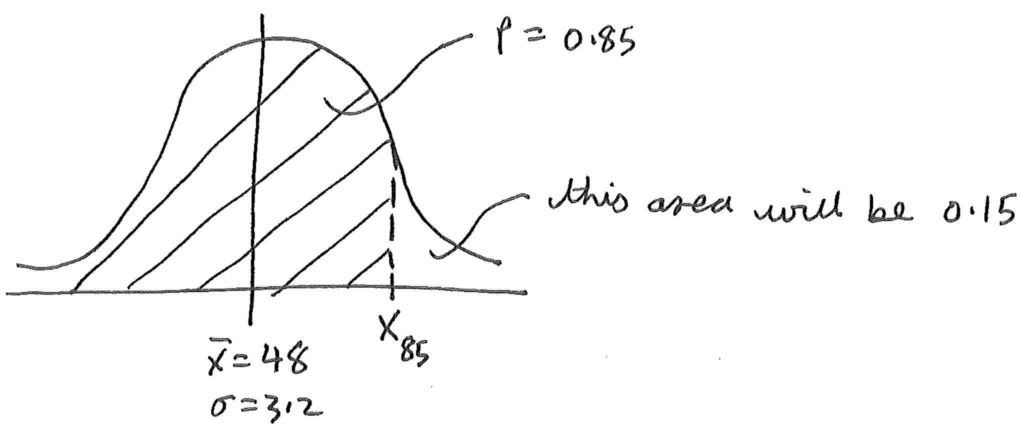
z: -3.6 $\quad\downarrow\quad$ p: $x = ?$

$$\frac{x - 0.0013}{-3.6 - (-3.0)} = \frac{0.0003 - 0.0013}{-3.4 - (-3.0)}$$

solving for x:

$$\boxed{x = 0.0002}$$ a very small probability

2.



So we look up 0.85 in the table and find associated z-score. You may need to interpolate if you cant find close value.

From table $0.8508 \longrightarrow z = 1.04$

so we must convert this z-score to an X value.

From General Formula

$$z = \frac{x - \bar{x}}{\sigma/\sqrt{n}}$$

so

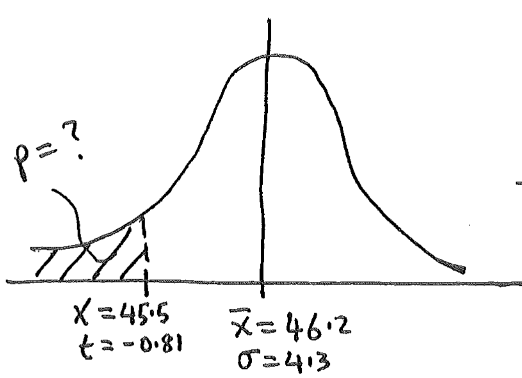
$$1.04 = \frac{X_{85} - 48}{3.2/\sqrt{60}} \Rightarrow \boxed{X_{85} = 48.42 \text{ mph}}$$

3) Now have a different sample with

$n = 25 < 30$ so we shall use t-distribution

So from General formula

$$t = \frac{x - \bar{x}}{\sigma/\sqrt{n}} = \frac{45.5 - 46.2}{4.3/\sqrt{25}} = -0.81$$



→ So we go to t-table and look at $n-1 = 24$ degrees of freedom.

Note that in t-table the t-values are in the table and the probabilities are in the headers, (opposite ~~from~~ of z-table). Also look at the diagram associated with the table. The shaded area is for a positive t-value. If we have a negetive t-value is simply the mirror image of the positive so we read the value from the table as-is, just noting that our t-value will have a -ve sign in front of it.

Now <u>t-table involves a lot of interpolation</u> so please be careful.

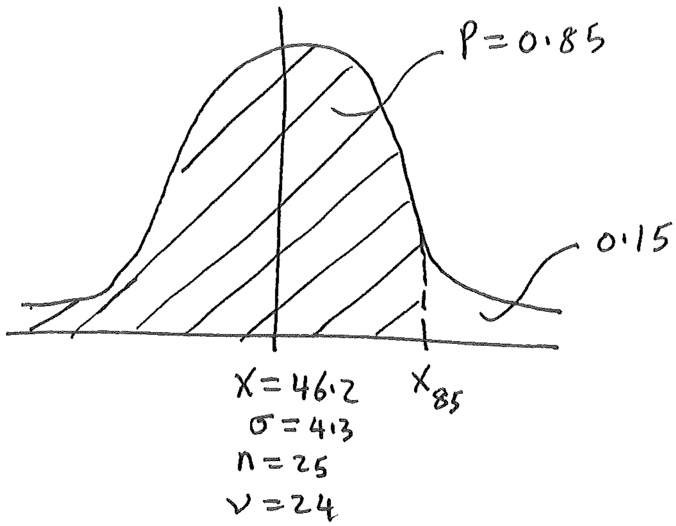
For 24 degrees of freedom:

t :	1.318	2.064	0.81
	↓	↓	↓
Prob. (α) :	0.1	0.025	$x = ?$

$$\frac{x - 0.1}{0.81 - 1.318} = \frac{0.1 - 0.025}{1.318 - 2.064} \Rightarrow x = 0.151$$

So $\boxed{P(X \leq 45.5) = 0.151}$

4)



Now due to the way our t-table is set (look at the picture) we have to find the t-value associated with 15% ! Because the t-table gives us the area under the bell curve from the tail.

so at 24 degrees of freedom :

α : 0.15 → 0.10 → 0.05

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t : x=? 1.318 1.711

$$\frac{x - 1.318}{0.15 - 0.10} = \frac{1.318 - 1.711}{0.10 - 0.05}$$

$x = 0.925$ so $t_{85} = 0.925$

so from general formula

$$t = \frac{x - \bar{x}}{\sigma/\sqrt{n}}$$

so

$$X_{85} = t_{85} \cdot \frac{\sigma}{\sqrt{n}} + \bar{x}$$

$$= \frac{0.925\,(4.3)}{\sqrt{25}} + 46.2$$

$$\boxed{X_{85} = 46.995 \text{ mph}}$$

Comments :

1. Note that if we were asked to find the 15th percentile we would basically run the same calculation but area under curve would have been on the left. So the t_{85} we calculated, we would simply stick a −ve sign in front of it before plugging into general formula to get the X value.

2. Note that different sample sizes yielded different result for the 85th percentile. Though both are technically correct, you want to go with the one with more data aka the bigger the n the better, if you can get the data.