comment on one factor that might influence your estimate.

3.29 When we roll a pair of balanced dice, what are the probabilities of getting

(a) 7;

(b) 11;

(c) 7 or 11;

(d) 3;

(e) 2 or 12;

(f) 2, 3, or 12?

3.30 A lottery sells tickets numbered from 00001 through 50000. What is the probability of drawing a number that is divisible by 200?

3.31 A car rental agency has 18 compact cars and 12 intermediate-size cars. If four of the cars are randomly selected for a safety check, what is the probability of getting two of each kind?

3.32 Among the first 842 convection ovens sold to consumers, 143 required some adjustment during the warranty period. Estimate the probability that a newly purchased convection oven will require some adjustment during the warranty period.

3.33 In a group of 160 graduate engineering students, 92 are enrolled in an advanced course in statistics, 63 are enrolled in a course in operations research, and 40 are enrolled in both. How many of these students are not enrolled in either course?

3.34 Among 150 persons interviewed as part of an urban mass transportation study, some live more than 3 miles from the center of the city (A), some now regularly drive their own car to work (B), and some would gladly switch to public mass transportation if it were available (C). Use the information given in Figure 3.10 to find

(a) $N(A)$;

(b) $N(B)$;

(c) $N(C)$;

(d) $N(A \cap B)$;

(e) $N(A \cap C)$;

(f) $N(A \cap B \cap C)$;

(g) $N(A \cup B)$;

(h) $N(B \cup C)$;

(i) $N(\overline{A} \cup \overline{B} \cup C)$;

(j) $N[B \cap (A \cup C)]$.

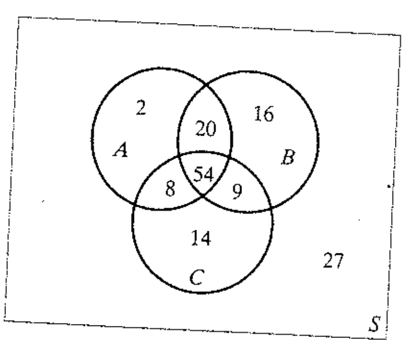

Figure 3.10 Diagram for Exercise 3.34

3.35 An experiment has the four possible mutually exclusive outcomes A, B, C, and D. Check whether the following assignments of probability are permissible:

(a) $P(A)=0.38$, $P(B)=0.16$, $P(C)=0.11$, $P(D)=0.35$;

(b) $P(A)=0.31$, $P(B)=0.27$, $P(C)=0.28$, $P(D)=0.16$;

(c) $P(A)=0.32$, $P(B)=0.27$, $P(C)=-0.06$, $P(D)=0.47$;

(d) $P(A)=\frac{1}{2}$, $P(B)=\frac{1}{4}$, $P(C)=\frac{1}{8}$, $P(D)=\frac{1}{16}$;

(e) $P(A)=\frac{5}{18}$, $P(B)=\frac{1}{6}$, $P(C)=\frac{1}{3}$, $P(D)=\frac{2}{9}$.

3.36 With reference to Exercise 3.1, suppose that the points (0, 0), (0, 1), (0, 2), (1, 0), (1, 1), (1, 2), (2, 0), (2, 1), (2, 2), (3, 0), (3, 1), and (3, 2) have the probabilities 0.060, 0.012, 0.006, 0.067, 0.014, 0.092, 0.260, 0.027, 0.080, 0.166, 0.110, and 0.106.

(a) Verify that this assignment of probabilities is permissible.

(b) Find the probabilities of events R, T, and U given in part (b) of that exercise.

(c) Calculate the probabilities that zero, one, or two streams are contaminated.

3.37 With reference to Exercise 3.7, suppose that each point (i, j) of the sample space is assigned the probability $\frac{15/28}{i+j}$.

(a) Verify that this assignment of probabilities is permissible.

(b) Find the probabilities of events B, C, and D described in part (b) of that exercise.

(c) Find the probabilities that one, two, or three of the graduate students will be supervising the physics lab.

3.38 Explain why there must be a mistake in each of the following statements:

(a) The probability that a mineral sample will contain silver is 0.38 and the probability that it will not contain silver is 0.52.

(b) The probability that a drilling operation will be a success is 0.34 and the probability that it will not be a success is -0.66.

(d) $P(\overline{A} \cup \overline{B})$.

(e) Are A and B independent?

3.90 In a sample of 446 cars stopped at a roadblock, only 67 of the drivers had their seatbelts fastened. Estimate the probability that a driver stopped on that road will have his or her seatbelt fastened.

3.91 The marketing manager reported to the head engineer regarding a survey concerning the company's portable cleaning tool. He claims that, among the 200 customers surveyed, 165 said the product is reliable, 117 said it is easy to use, 88 said it is both reliable and easy to use, and 33 said it is neither reliable nor easy to use. Explain why the head engineer should question this claim.

3.92 If the probabilities that a satellite launching rocket will explode during lift-off or have its guidance system fail in flight are 0.0002 and 0.0005, find the probabilities that such a rocket will

(a) not explode during lift-off;

(b) explode during lift-off or have its guidance system fail in flight;

(c) neither explode during lift-off nor have its guidance system fail in flight.

3.93 Given $P(A) = 0.20$, $P(B) = 0.45$, and $P(A \cap B) = 0.09$, verify that

(a) $P(A \mid B) = P(A)$;

(b) $P(A \mid \overline{B}) = P(A)$;

(c) $P(B \mid A) = P(B)$;

(d) $P(B \mid \overline{A}) = P(B)$.

3.94 If events A and B are independent and $P(A) = 0.45$ and $P(B) = 0.30$, find

(a) $P(A \cap B)$;

(b) $P(A \mid B)$;

(c) $P(A \cup B)$;

(d) $P(\overline{A} \cap \overline{B})$.

3.95 The following frequency table shows the classification of 58 landfills in a state according to their concentration of the three hazardous chemicals arsenic, barium, and mercury.

		Barium			
		High		Low	
		Mercury		Mercury	
		High	Low	High	Low
Arsenic	High	1	3	5	9
	Low	4	8	10	18

If a landfill is selected at random, find the probability that it has

(a) a high concentration of mercury;

(b) a high concentration of barium and low concentrations of arsenic and mercury;

(c) high concentrations of any two of the chemicals and low concentration of the third;

(d) a high concentration of any one of the chemicals and low concentrations of the other two.

3.96 Refer to Exercise 3.95. Given that a landfill, selected at random, is found to have a high concentration of barium, what is the probability that its concentration is

(a) high in mercury?

(b) low in both arsenic and mercury?

(c) high in either arsenic or mercury?

3.97 An explosion in an LNG storage tank in the process of being repaired could have occurred as the result of static electricity, malfunctioning electrical equipment, an open flame in contact with the liner, or purposeful action (industrial sabotage). Interviews with engineers who were analyzing the risks involved led to estimates that such a explosion would occur with probability 0.25 as a result of static electricity, 0.20 as a result of malfunctioning electric equipment, 0.40 as a result of an open flame, and 0.75 as a result of purposeful action. These interviews also yielded subjective estimates of the prior probabilities of these four causes of 0.30, 0.40, 0.15, and 0.15, respectively. What was the most likely cause of the explosion?

3.98 An engineer wants to move across the country from City A to City B to work. On different days, she mails a cover letter and resume to three different companies in City B. Suppose that a letter sent from City A to City B has a probability of 0.7 of reaching City B within 3 days.

(a) What is the probability that exactly 2 of the 3 letters will reach City B within 3 days?

(b) If the three letters are mailed together at the same time and location, how does your conclusion in part (a) change? Explain.

3.99 Amy commutes to work by two different routes A and B. If she comes home by route A, then she will be home no later than 6 P.M. with probability 0.8, but if she comes home by route B, then she will be home no later than 6 P.M. with probability 0.7. In the past, the proportion of times that Amy chose route A is 0.4.

(a) What proportion of times is Amy home no later than 6 P.M.?

(b) If Amy is home after 6 P.M. today, what is the probability that she took route B?

3.78 Two firms V and W consider bidding on a road-building job, which may or may not be awarded depending on the amounts of the bids. Firm V submits a bid and the probability is $\frac{3}{4}$ that it will get the job provided firm W does not bid. The probability is $\frac{3}{4}$ that W will bid, and if it does, the probability that V will get the job is only $\frac{1}{3}$.

(a) What is the probability that V will get the job?

(b) If V gets the job, what is the probability that W did not bid?

3.79 Engineers in charge of maintaining our nuclear fleet must continually check for corrosion inside the pipes that are part of the cooling systems. The inside condition of the pipes cannot be observed directly but a nondestructive test can give an indication of possible corrosion. This test is not infallible. The test has probability 0.7 of detecting corrosion when it is present but it also has probability 0.2 of falsely indicating internal corrosion. Suppose the probability that any section of pipe has internal corrosion is 0.1.

(a) Determine the probability that a section of pipe has internal corrosion, given that the test indicates its presence.

(b) Determine the probability that a section of pipe has internal corrosion, given that the test is negative.

3.80 An East Coast manufacturer of printed circuit boards exposes all finished boards to an online automated verification test. During one period, 900 boards were completed and 890 passed the test. The test is not infallible. Of 30 boards intentionally made to have noticeable defects, 25 were detected by the test. Use the relative frequencies to approximate the conditional probabilities needed below.

(a) Give an approximate value for P[Pass test | board has defects].

(b) Explain why your answer in part a may be too small.

(c) Give an approximate value for the probability that a manufactured board will have defects. In order to answer the question, you need information about the conditional probability that a good board will fail the test. This is important to know but was not available at the time an answer was required. To proceed, you can assume that this probability is zero.

(d) Approximate the probability that a board has defects given that it passed the automated test.

Do's and Don'ts

Do's

1. Begin by creating a sample space S which specifies all possible outcomes.
2. Always assign probabilities to events that satisfy the axioms of probability. In the discrete case, the possible outcomes can be arranged in a sequence. The axioms are then automatically satisfied when probability p_i is assigned to the ith outcome, where

$$0 \le p_i \qquad \text{and} \qquad \sum_{\text{all outcomes in } S} p_i = 1$$

and the probability of any event A is defined as

$$P(A) = \sum_{\text{all outcomes in } A} p_i$$

3. Combine the probabilities of events according to rules of probability.

General Addition Rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Rule of the Complement: $P(\overline{A}) = 1 - P(A)$

General Multiplication Rule: $P(A \cap B) = P(A)P(B \mid A)$ if $P(A) \neq 0$
$= P(B)P(A \mid B)$ if $P(B) \neq 0$

Conditional Probability: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ if $P(B) \neq 0$