Center of Gravity / Centroid

9-42

$$\text{Volume} = \left(\begin{matrix}\text{cross section} \\ \text{area}\end{matrix}\right)(\text{length})$$

so for an infinitesimally small volume

$$dV = \frac{1}{2}\pi r^2 \cdot dy$$

in this case radius r, is the z value so

$$dV = \frac{1}{2}\pi z^2 dy$$

we are told that

$$z = \frac{a}{h} y$$

so

$$dV = \frac{\pi}{2}\left(\frac{a}{h} y\right)^2 dy$$

$$dV = \frac{\pi a^2}{2h^2} y^2 dy$$

so can integrate for all y.

$$\int_0^V dV = \int_0^h \frac{\pi a^2}{2h^2} y^2 dy$$

$$V = \frac{\pi^2 a}{2h^2} \int_0^h y^2 dy$$