

$$v = \frac{\pi a^2}{2h^2} \left[ \frac{y^3}{3} \right]_0^h$$

$$= \frac{\pi a^2 h^3}{2h^2 \cdot 3}$$

$$= \frac{\pi a^2 h^2}{3}$$

$$= \frac{\pi a^2 h}{6}$$

centroid :

$$\bar{y} = \frac{\int_v y \, dv}{\int_v dv}$$

$$= \frac{\int_0^h \frac{\pi a^2}{2h^2} y \cdot y^2 \, dy}{\int_0^h dv} = \frac{\frac{\pi a^2}{2h^2} \int_0^h y^3 \, dy}{\pi a^2 h / 6}$$

$$= \frac{\frac{\pi a^2}{2h^2} \left[ \frac{y^4}{4} \right]_0^h}{\pi a^2 h / 6} = \frac{\frac{\pi a^2}{8h^2} \cdot h^4}{\pi a^2 h / 6} = \frac{3}{4} h$$