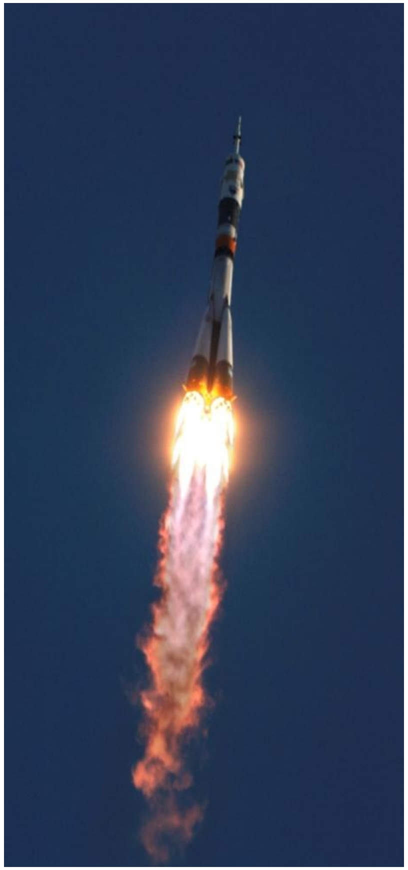


Force Vectors

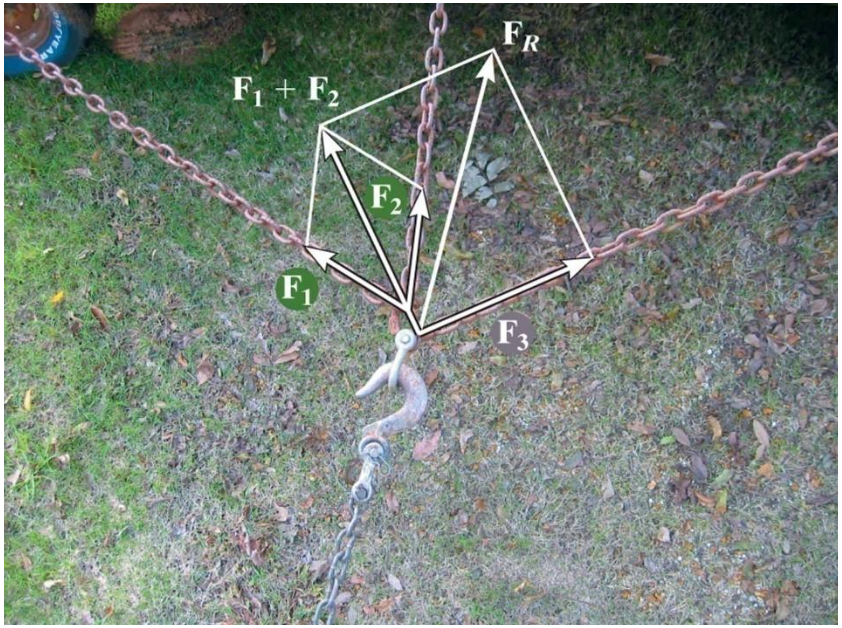
Chapter 2

Overview

- Vectors
- Vector Operations
- Vector Addition of Forces
- Coplanar Forces
- Cartesian Vectors
- Position Vectors & Force Vectors
- Dot Product

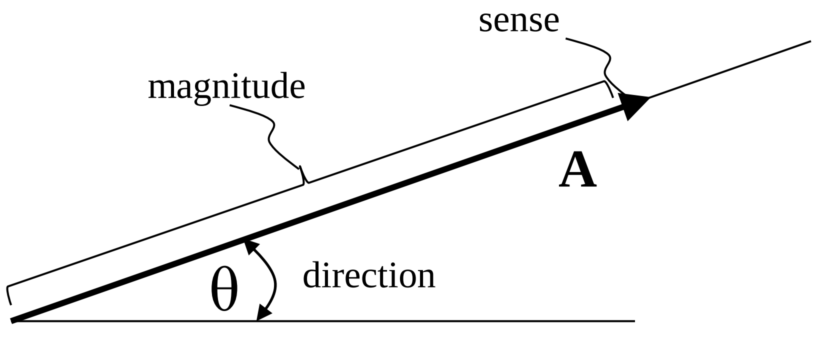
Force Vectors

- The three chains pulling on the hook are exerting three forces on it.



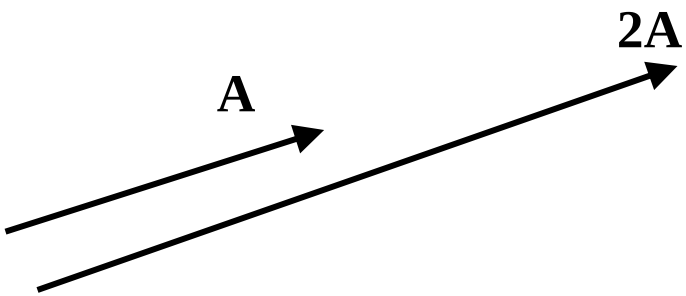
Scalars & Vectors

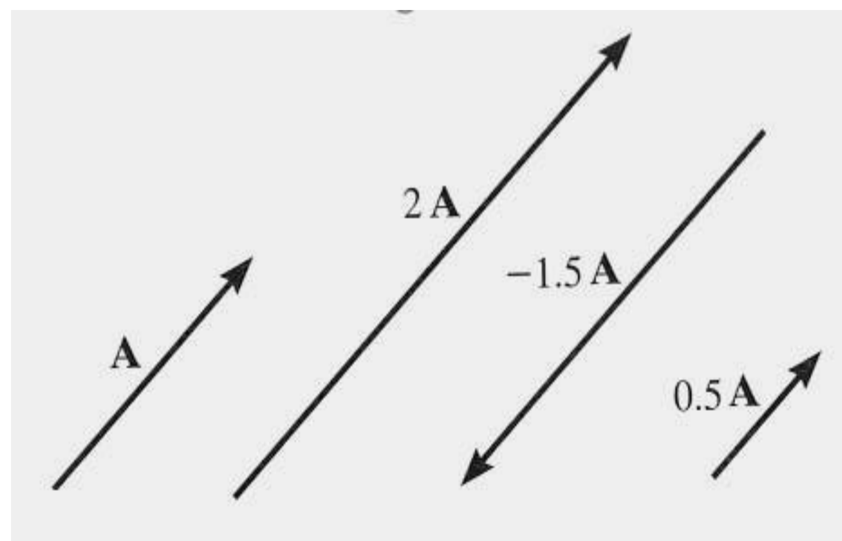
- Scalar: This is any positive or negative physical quantity characterized completely by its magnitude
- Vector: A physical quantity that is completely described by a magnitude and a direction
- Vector notation: **A** or $\vec{A}$



Vector Operations

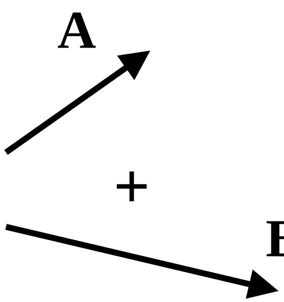
- Scalar Multiplication and Division

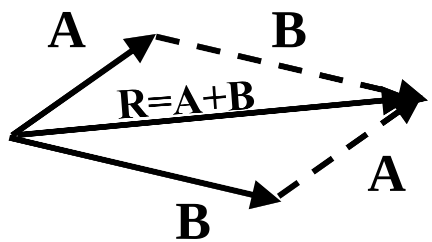




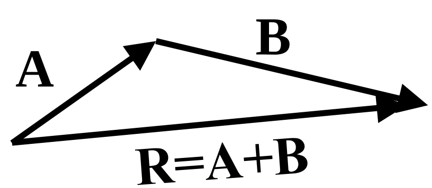
Vector Addition

- Parallelogram law

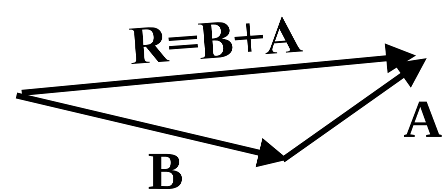




- Triangle method



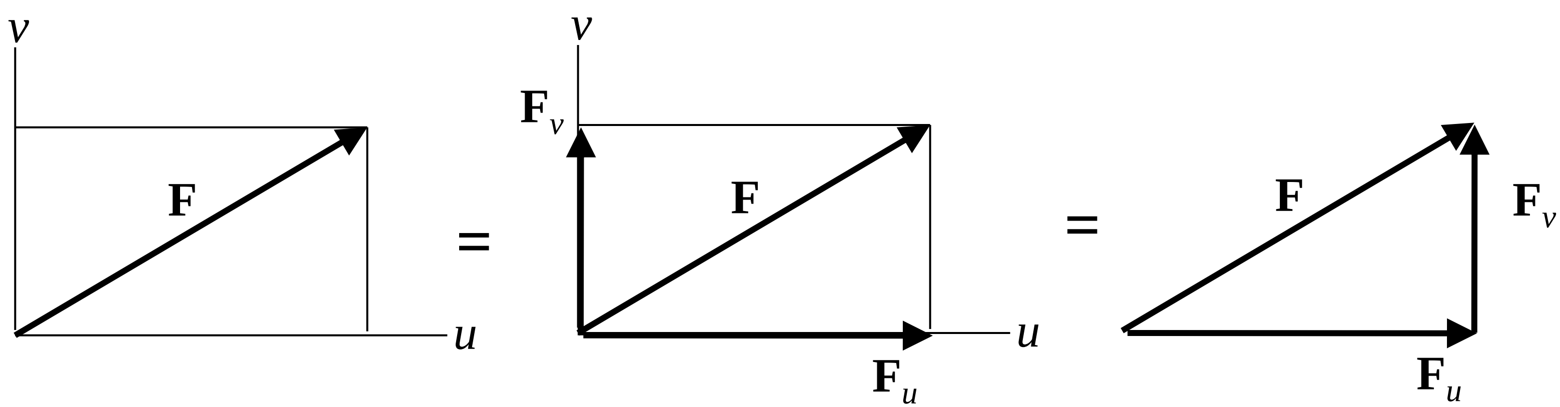
OR



- The combined effect of **A** and **B**, **R,** is called the *resultant* vector
- How would you do **A** – **B** ?

Resolution of a Vector

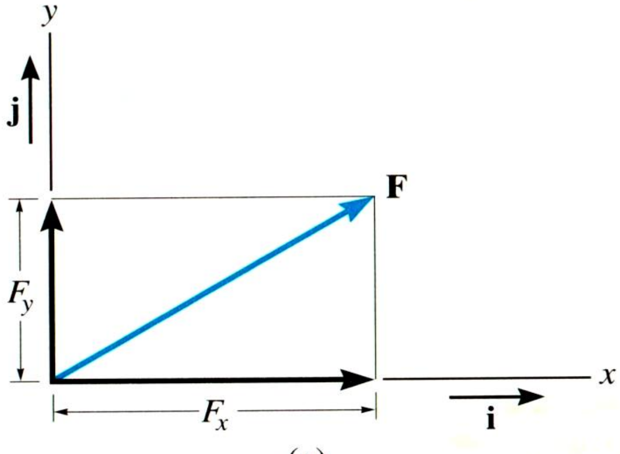
- This is breaking up a vector into components along given axes or resolving the vector



Addition of Coplanar Forces

- We can resolve the components of our vector in the *x* and *y* axes, with their respective magnitudes
- ***i*** and ***j*** represent *unit vectors* in *x* and *y* directions respectively
- We can therefore represent the force vector *F* as

  $\boldsymbol{F} = F_x\,\boldsymbol{i} + F_y\,\boldsymbol{j}$

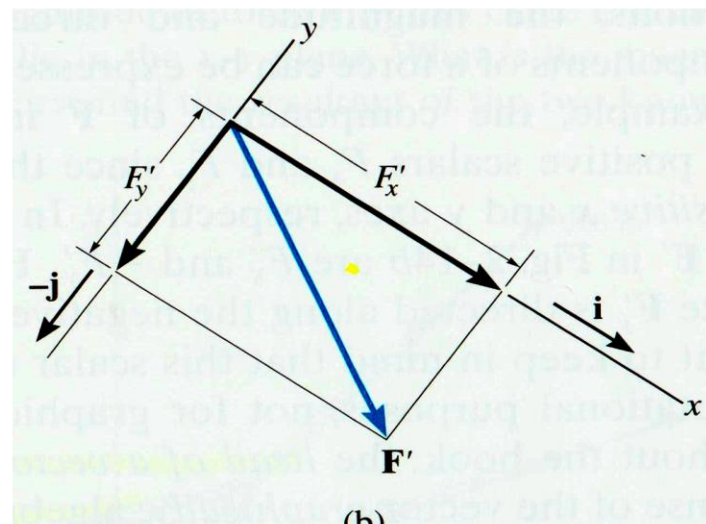


(a)

Addition of Coplanar Force Vectors

- The *x* and *y* axes are always perpendicular to each other.
- However, they can be directed at any inclination.
- In this example we can express the force vector as

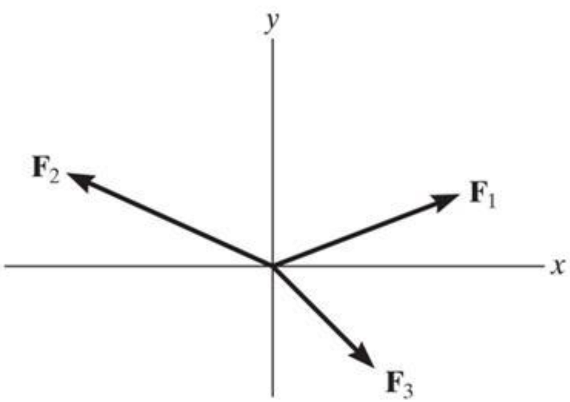
  $\boldsymbol{F} = F'_x\,\boldsymbol{i} + F'_y\,\boldsymbol{j}$

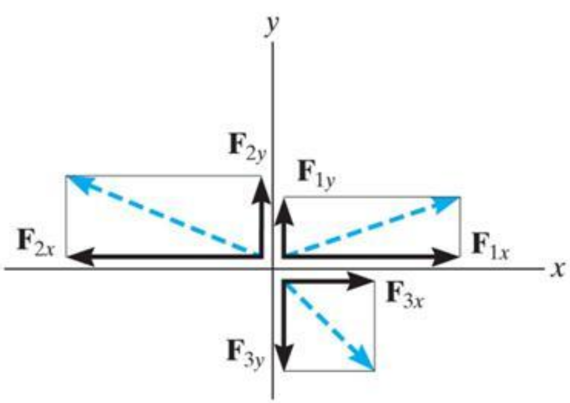


Addition of Several Vectors

- We can use the resolved components to add several vectors by summing up corresponding components

  » Step 1: is to resolve each force into its components





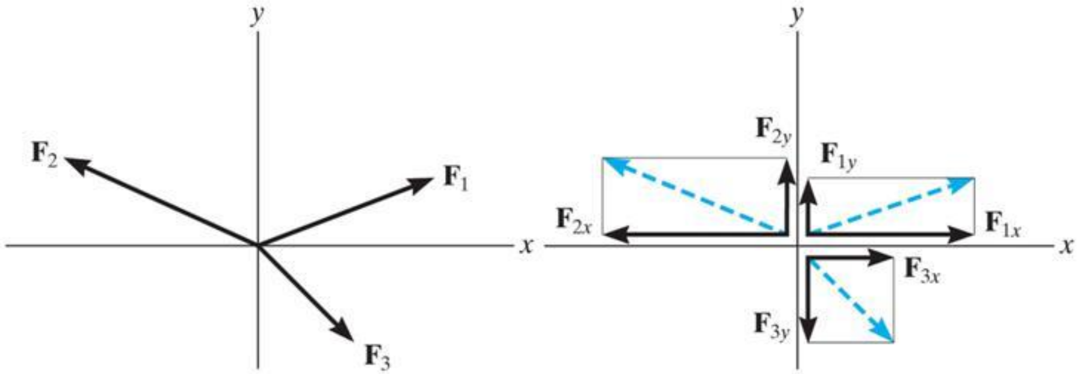
Step 2: add all the x-components together. Add all the y-components. These two totals are the *x* and *y* components of the resultant vector.

Step 3 is to find the magnitude and angle of the resultant vector.

Addition of Several Vectors

An illustration of the process

- Break the three vectors into components, then add them

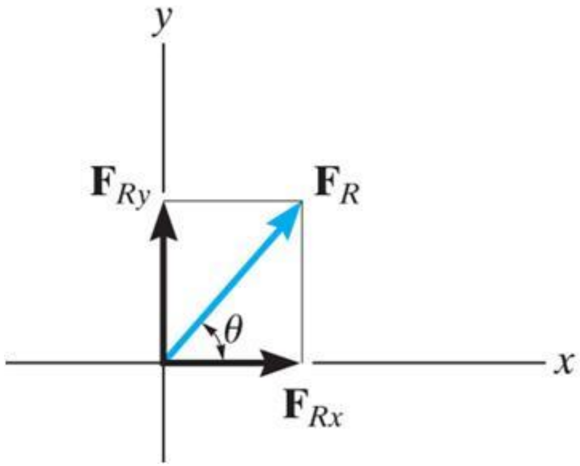


- $\boldsymbol{F_R} = \boldsymbol{F_1} + \boldsymbol{F_2} + \boldsymbol{F_3}$
  $= F_{1x}\,\boldsymbol{i} + F_{1y}\,\boldsymbol{j} - F_{2x}\,\boldsymbol{i} + F_{2y}\,\boldsymbol{j} + F_{3x}\,\boldsymbol{i} - F_{3y}\,\boldsymbol{j}$
  $= (F_{1x} - F_{2x} + F_{3x})\,\boldsymbol{i} + (F_{1y} + F_{2y} - F_{3y})\,\boldsymbol{j}$
  $= (F_{Rx})\,\boldsymbol{i} + (F_{Ry})\,\boldsymbol{j}$

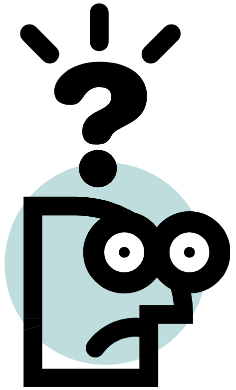
Coplanar Force Vectors

- How would you do the previous example using Triangle method or Parallelogram law ??
- We can also represent a 2-D vector with a magnitude and angle

- $F_{Ry} = F_R \sin\theta$
- $F_{Rx} = F_R \cos\theta$
- $\theta = \tan^{-1}(F_{Ry} / F_{Rx})$
- $F_R = \text{SQRT}\{(F_{Ry})^2 + (F_{Ry})^2\}$

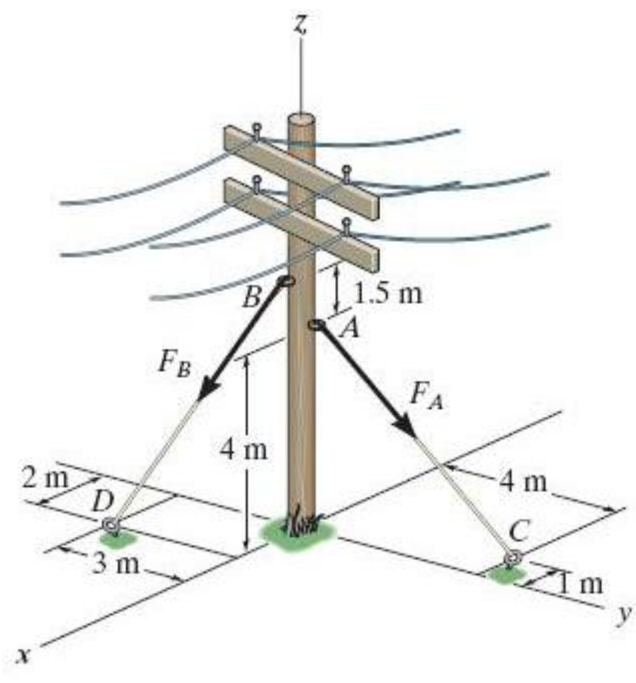


Questions & Comments ??



Cartesian Vectors

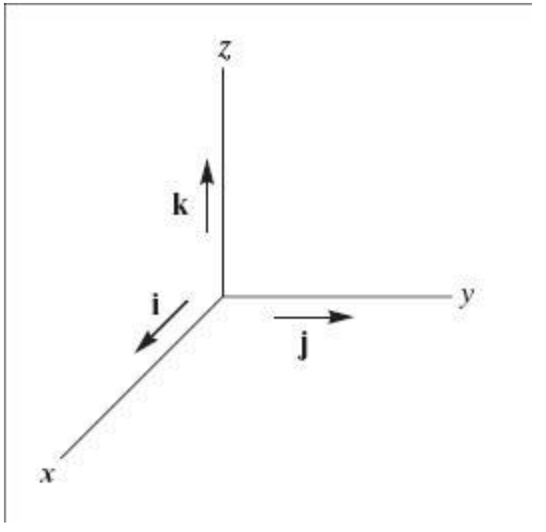
- Solving Vector problems can be simplified if we represent the vectors in *Cartesian vector form*
- This is particularly the case for 3-D problems



Cartesian Coordinates

- For a vector $\boldsymbol{A}$, with a magnitude of A, an unit vector is defined as

$$\boldsymbol{u}_A = \boldsymbol{A} / A .$$

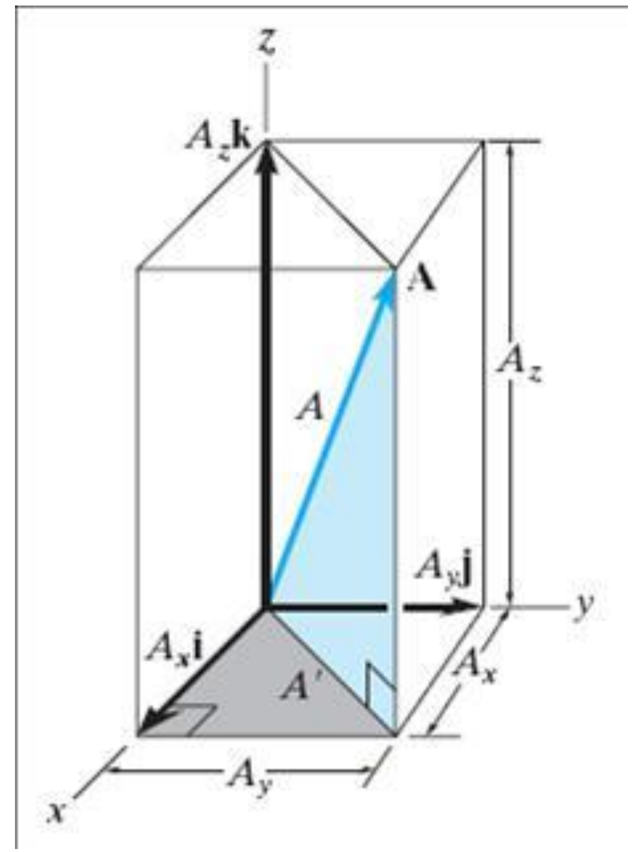


Properties of a unit vector

- Its magnitude is 1
- It is *dimensionless* (no units).
- It points in the same direction as the original vector ($\boldsymbol{A}$).
- in the Cartesian axis system $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ unit vectors along the positive x, y, and z axes respectively.

Cartesian Vector Representation

- In this example, the vector A can be defined as
  A = (A_X ***i*** **+** A_Y ***j*** **+** A_Z ***k*)**
- The *projection* of vector **A** in the x-y plane is A′.
- The magnitude of A' is
  $|A'| = (A_X^2 + A_Y^2)^{1/2}$
- The magnitude of the position vector **A** is therefore
  A $= ((A')^2 + A_Z^2)^{½}$
  $= (A_X^2 + A_Y^2 + A_Z^2)^{½}$



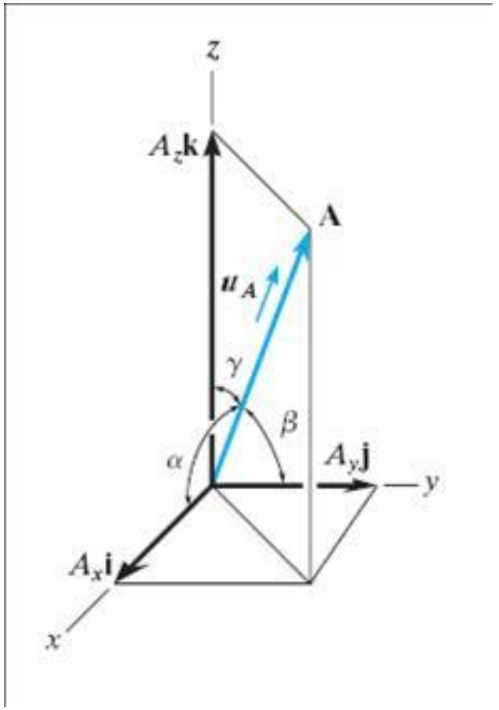
Direction of a Cartesian Vector

- The direction or orientation of vector A is defined by the angles ά, β, and γ.
- Using trigonometry, *direction cosines* are found using

$$\cos\alpha = \frac{A_x}{A} \qquad \cos\beta = \frac{A_y}{A} \qquad \cos\gamma = \frac{A_z}{A}$$

where

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$



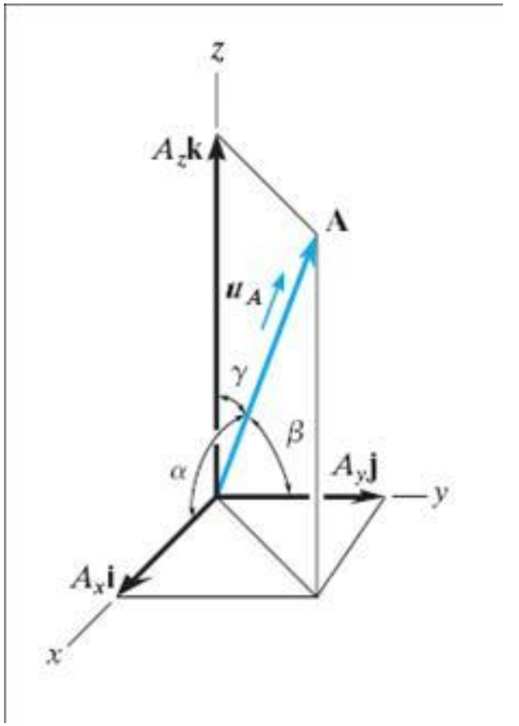
Direction of a Cartesian Vector

- Recall, the unit vector is defined by

$$\mathbf{u}_A = \frac{\mathbf{A}}{A} = \frac{A_x}{A}\,\mathbf{i} + \frac{A_y}{A}\,\mathbf{j} + \frac{A_z}{A}\,\mathbf{k}$$

- Which can be rewritten as

$$u_A = \cos\alpha\,\boldsymbol{i} + \cos\beta\,\boldsymbol{j} + \cos\gamma\,\boldsymbol{k}$$



Addition of Cartesian Vectors

- Extending the concept of 2-D vector addition

$$\mathbf{F}_R = \Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$$

- So that if $\mathbf{A} = A_X\,\boldsymbol{i} + A_Y\,\boldsymbol{j} + A_Z\,\boldsymbol{k}$ and
  $\mathbf{B} = B_X\,\boldsymbol{i} + B_Y\,\boldsymbol{j} + B_Z\,\boldsymbol{k}$, then

- $\mathbf{A+B} = (A_X + B_X)\boldsymbol{i} + (A_Y + B_Y)\boldsymbol{j} + (A_Z + B_Z)\,\boldsymbol{k}$

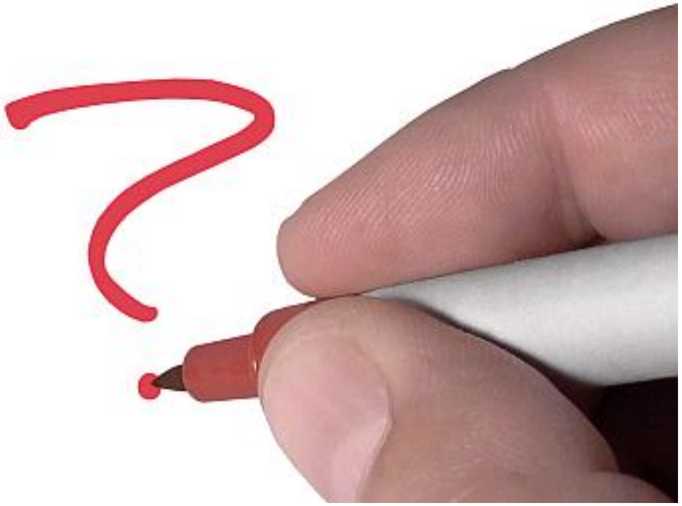
and

- $\mathbf{A-B} = (A_X - B_X)\boldsymbol{i} + (A_Y - B_Y)\boldsymbol{j} + (A_Z - B_Z)\,\boldsymbol{k}$

Problem Solving Tips

- You may be given 3-D vector information as:
  - Magnitude and the coordinate direction angles, or,
  - Magnitude and projection angles.
- It helps to change the representation of the vector into the Cartesian form

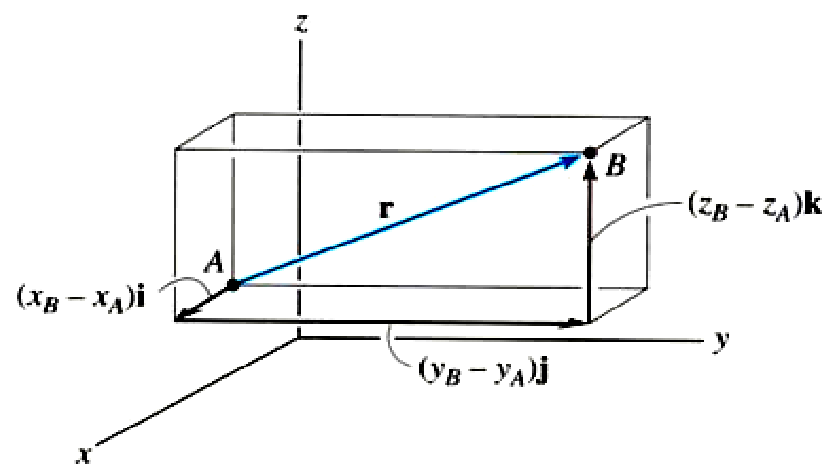
Questions & Comments ?



Position Vectors and Force Vectors

Position Vectors

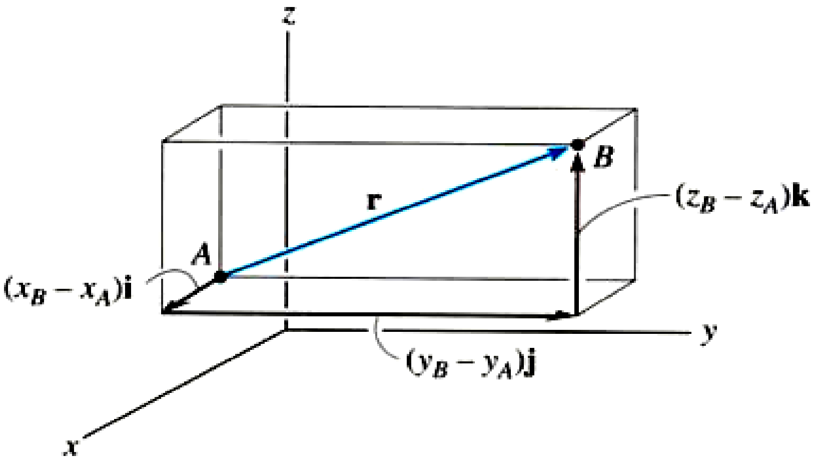
- A position vector is defined as a fixed vector that locates a point in space relative to another point
- A and B have coordinates be (X_A, Y_A, Z_A) and (X_B, Y_B, Z_B), respectively.



Position Vectors

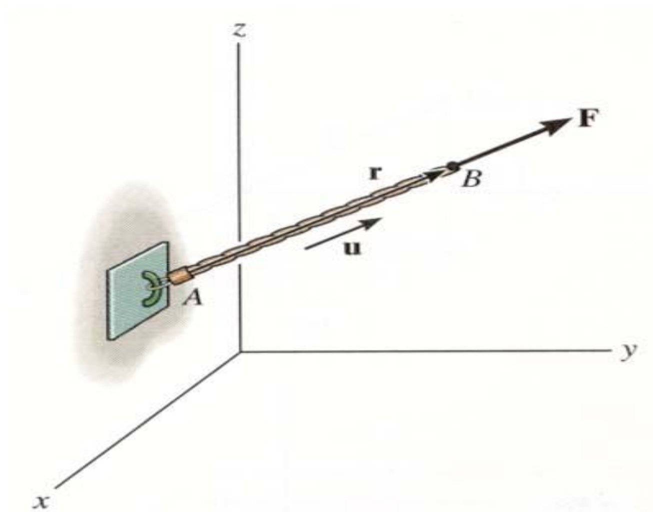
- The position vector from A to B, $\boldsymbol{r}_{AB}$, is defined as

$$\boldsymbol{r}_{AB} = \{(X_B - X_A)\boldsymbol{i} + (Y_B - Y_A)\boldsymbol{j} + (Z_B - Z_A)\,\boldsymbol{k}\}\mathrm{m}$$



Force Vector

- If a force is directed along a line, then we can represent the force vector in Cartesian coordinates by using a unit vector and the force's magnitude.
- position vector is $\boldsymbol{r}_{AB}$, along two points on that line.
- unit vector describing the line's direction is
  $\boldsymbol{u}_{AB} = (\boldsymbol{r}_{AB}/r_{AB})$
- Force vector is magnitude of force times the unit vector
  $\boldsymbol{F} = F\ \boldsymbol{u}_{AB}$



Questions & Comments

Dot Product

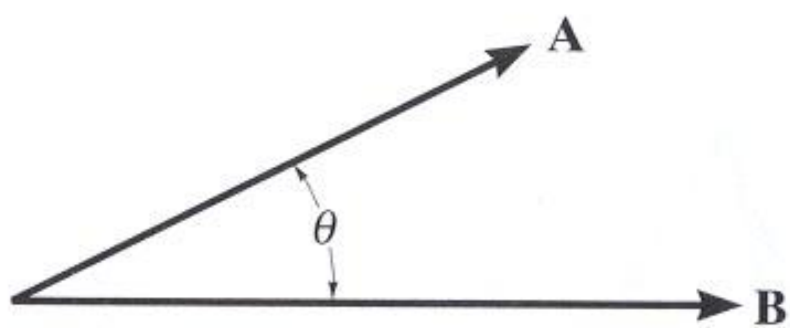
Definition:

The dot product of vectors ***A*** and ***B*** is defined as $\boldsymbol{A}\bullet\boldsymbol{B} = |A|\ |B| \cos\theta$.

Where $\boldsymbol{A}\bullet\boldsymbol{B} = (A_x\boldsymbol{i} + A_y\boldsymbol{j} + A_z\boldsymbol{k})\bullet(B_x\boldsymbol{i} + B_y\boldsymbol{j} + B_z\boldsymbol{k})$

$= A_x B_x + A_y B_y + A_z B_z$

The angle θ is the smallest angle between the two vectors and is always in a range of 0° to 180°.



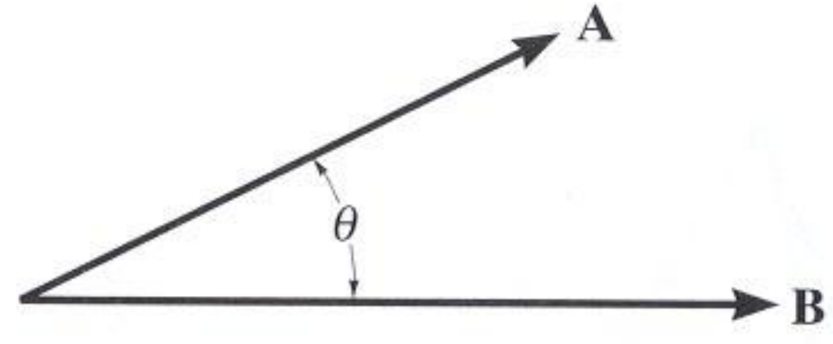
Dot Product

Properties

- The result of the dot product is a scalar (a positive or negative number).
- The units of the dot product will be the product of the units of the ***A*** and ***B*** vectors
- By definition, $\boldsymbol{i} \bullet \boldsymbol{j} = 0$ and
  $\boldsymbol{i} \bullet \boldsymbol{i} = 1$

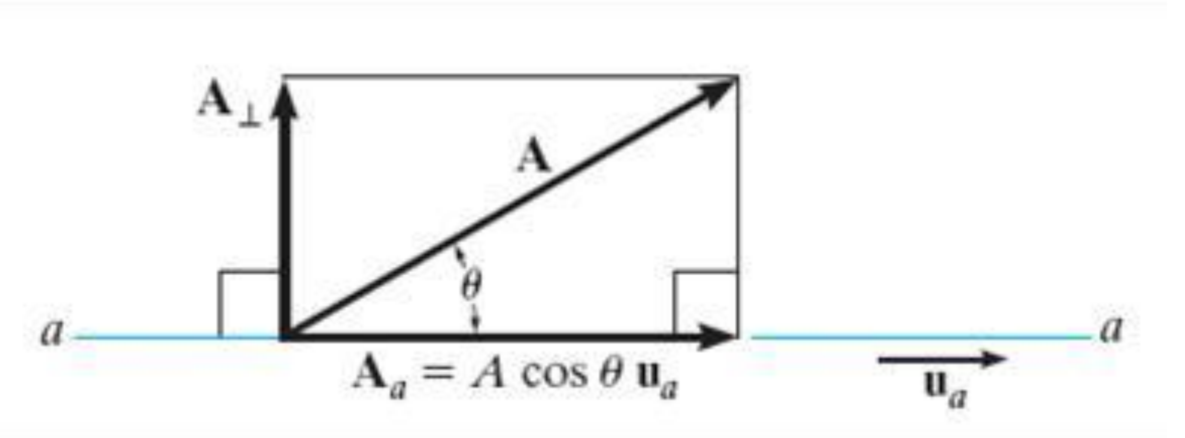
Dot Product

- For two known vectors we can use the dot product to find the angle between the them
- θ = $\cos^{-1}$ [$(\boldsymbol{A \bullet B})/(|A|\ |B|)$],
  where $0^{\circ} \leq \theta \leq 180^{\circ}$



Dot Product

- The dot product can be used to determine the *projection of a vector* parallel and perpendicular to a line aka components
- Step 1: Find the unit vector, $\boldsymbol{u}_{aa'}$ along line aa′
- Step 2: Find the scalar projection of ***A*** along line aa′ by $A_{||} = \boldsymbol{A} \bullet \boldsymbol{u}_{aa} = A_x U_x + A_y U_y + A_z U_z$



Dot Product

- Step 3: the projection can also be written as vector, $\boldsymbol{A}_{//}$, by using the unit vector $\boldsymbol{u}_{aa'}$ and the magnitude found in step 2.

$$\boldsymbol{A}_{//} = A_{||}\, \boldsymbol{u}_{aa'}$$

- The scalar and vector forms of the perpendicular component can easily be obtained by

$A_{\perp} = (A^2 - A_{||}^2)^{½}$ and

$\boldsymbol{A}_{\perp} = \boldsymbol{A} - \boldsymbol{A}_{//}$

or $\boldsymbol{A} = \boldsymbol{A}_{\perp} + \boldsymbol{A}_{//}$

Questions & Comments