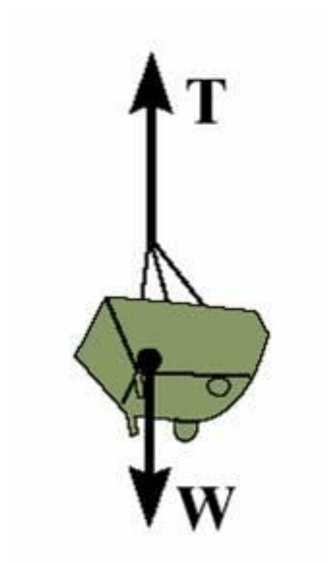
Equilibrium of a Particle

Chapter 3

Overview

- Equilibrium
- Free body diagram (FBD)
- Coplanar Forces
- Equation of Two-Dimensional Equilibrium
- Three-Dimensional Force Systems

Examples



Definition of Equilibrium of a Particle

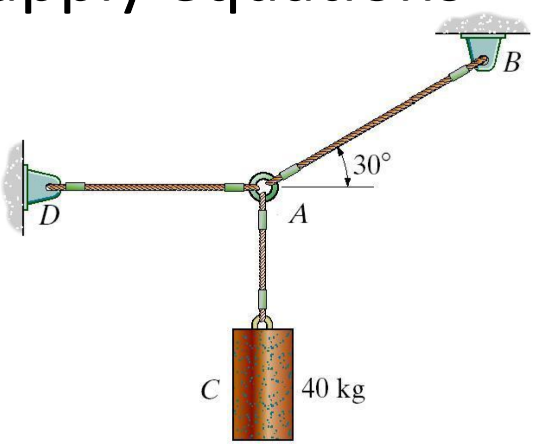
- A particle is said to be in *equilibrium* if it remains at rest, or has constant velocity if originally in motion.
- Mathematically,

$$\sum F = 0$$

- In other words the sum of all forces acting on the particle sum to zero. This is a corollary of Newton's 2^{nd} Law, $\sum F = ma$ where acceleration $a = 0$

COPLANAR FORCE SYSTEMS

- Consider the system shown
- The forces are restricted to a two-dimensional frame of reference (2-D)
- Such a 2-D system is commonly referred to as a *coplanar* system
- To determine the tensions in the cables we need to draw a *free body diagram* and then apply equations of equilibrium.



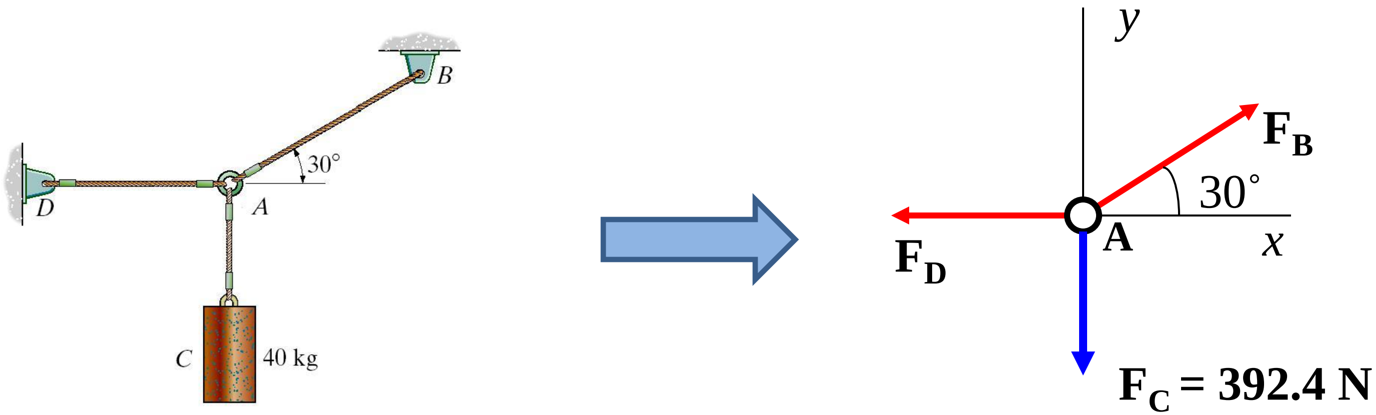
FREE BODY DIAGRAM (FBD)

- A drawing that shows all external forces acting on the particle.
- It is essentially a declaration of the forces in play
- It is key to being able to write the equations of equilibrium—which will be used to solve for the unknown forces and/or angles
- You MUST understand this concept to make progress in mechanics

Free Body Diagram

- Consider the particle A of this system were to be to be isolated from its surroundings.
- What forces would act on A?
- Active forces: trying to move A
- Reactive forces: resisting the motion
- Identify each force and show all known magnitudes and directions.
- Show the unknown forces' magnitudes and directions as variables

Free Body Diagram



- Note that we can change our perspective and draw the FDB with respect to say B, C or D.
- Typically, select the point that will best help in solving the problem at hand

EQUATIONS OF 2-D EQUILIBRIUM

- We can now the use the free body diagram to assist us perform the analysis
- Since particle A is in equilibrium, the net force at A is zero. This implies

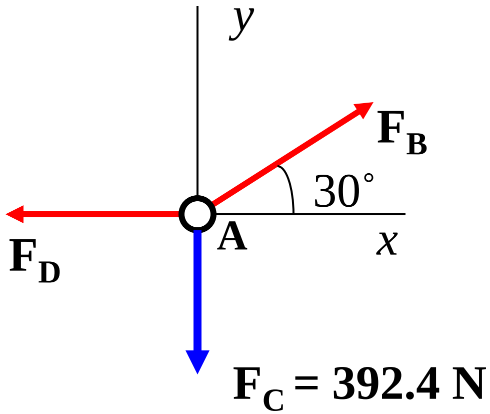
$$F_B + F_C + F_D = 0 \quad \text{or} \quad \Sigma F = 0$$

- In general, for a particle in eqiuiibrium;

$$\Sigma F = 0 \quad \text{or}$$

$$\Sigma F_x i + \Sigma F_y j = 0 = 0 i + 0 j$$

(vector formulation)

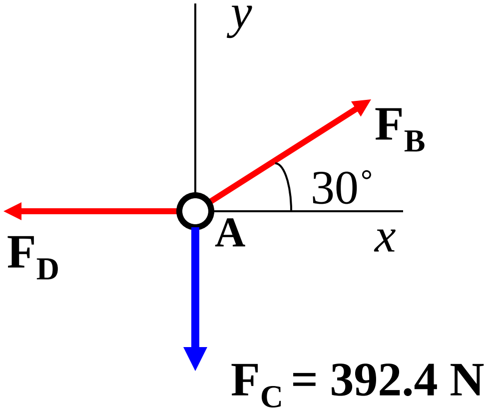


EQUATIONS OF 2-D EQUILIBRIUM

- Or we can express as follows, written in a scalar form,

$\Sigma F_x = 0$ and $\Sigma F_y = 0$ (scalar formulation)

- These are two scalar equations of equilibrium
- We can use these two scalar equations to solve for up to two unknowns.

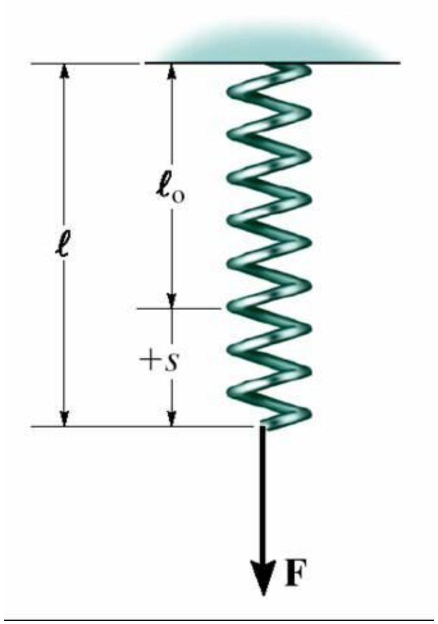


SPRINGS, CABLES, AND PULLEYS

- These present special cases encountered in engineering
- Consider a spring stretched from its original position
- Spring Force = spring constant * elongation

  or

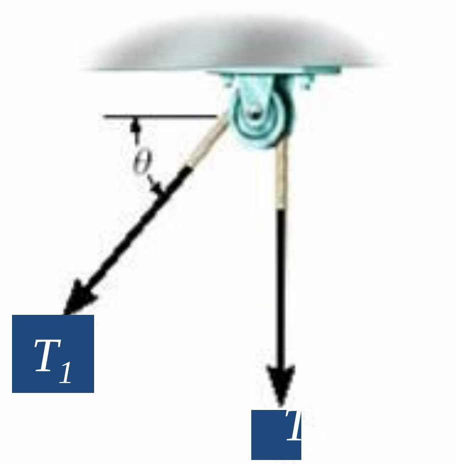
- F = k * s



SPRINGS, CABLES, AND PULLEYS

- Consider a pulley.
- Assuming friction is zero,
- $T_1 = T_2$

- We can take advantage of these special cases when solving problems



Cable is in tension

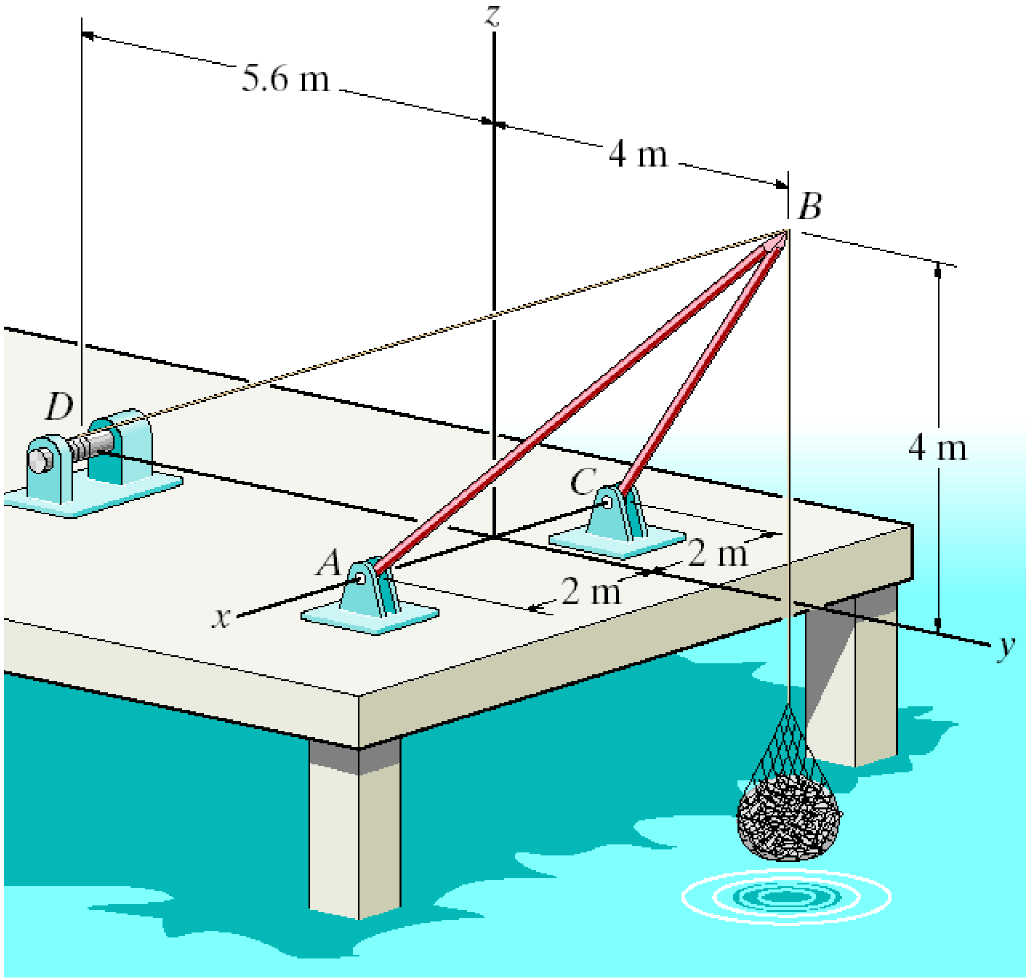
QUESTIONS & COMMENTS

THREE-DIMENSIONAL FORCE SYSTEMS

Overview:

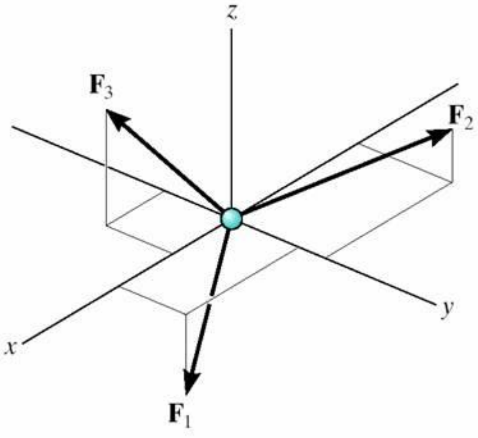
- 3-D free body diagram
- Applying the three scalar equations of equilibrium
- 3-D particle equilibrium problems

EXAMPLE



THE EQUATIONS OF 3-D EQUILIBRIUM

- For a particle in equilibrium, the vector sum of all the forces acting on it must be zero ($\Sigma \boldsymbol{F} = 0$) .
- For the x, y, and z components $(\Sigma F_x)\,\boldsymbol{i} + (\Sigma F_y)\,\boldsymbol{j} + (\Sigma F_z)\,\boldsymbol{k} = 0$
- Which implies: $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma F_z = 0$
- These equations are the *three scalar equations of equilibrium*
- The above enable us to solve for up to three unknowns.



QUESTIONS & COMMENTS

- Solve problems