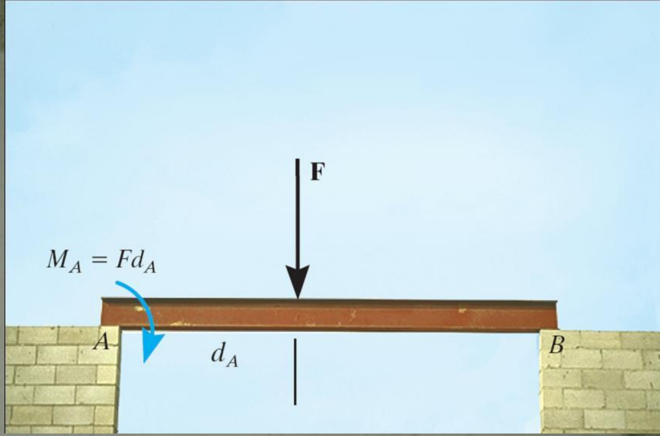
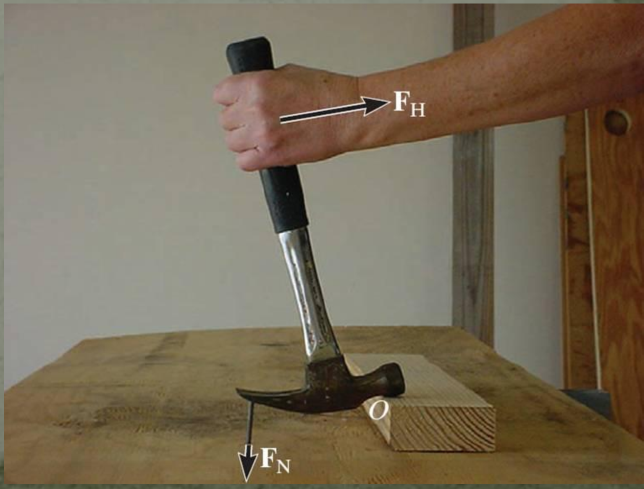


Force System Resultants

Chapter 4

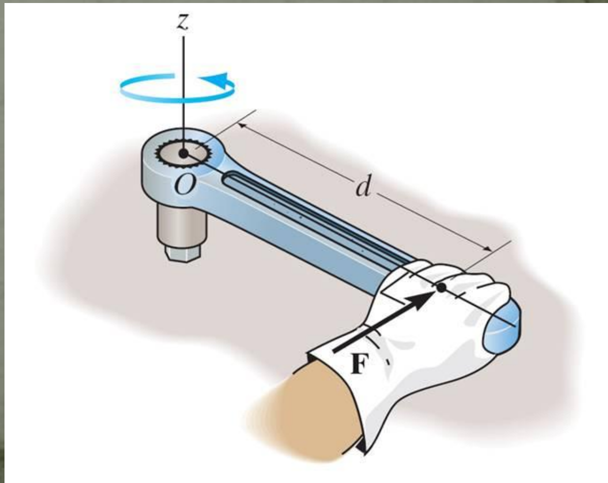
Overview

- Moment of a force (scalar, vector)
- Cross product
- Principle of Moments
- Couples
- Force and Couple Systems
- Simple Distributed Loading



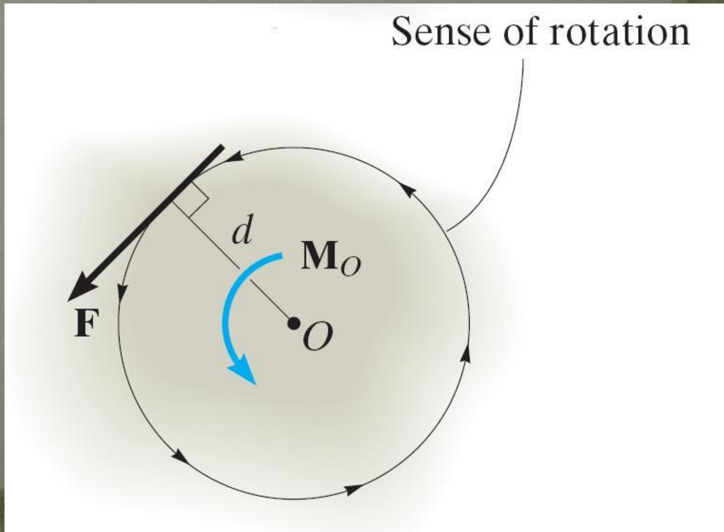
Moment of a force (scalar formulation)

- The *moment* of a force about a point provides a measure of the tendency for rotation (sometimes called a *torque*).
- the *magnitude* of the moment is $M_o = F\, d$
- d is the *perpendicular* distance from point O to the *line of action* of the force.



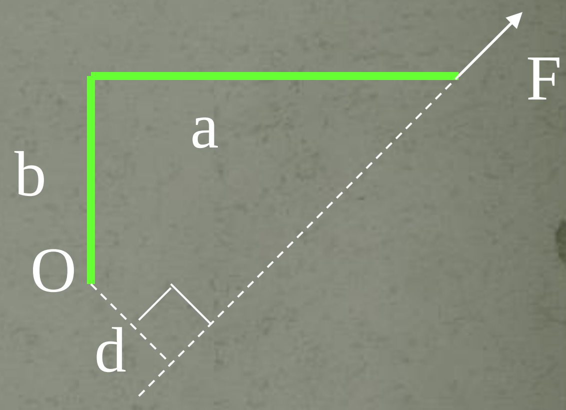
Moment of a force

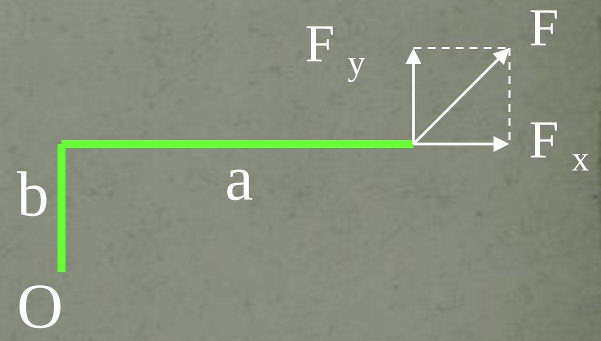
- the *direction* of M_O is either clockwise or counter-clockwise, depending on the tendency for rotation

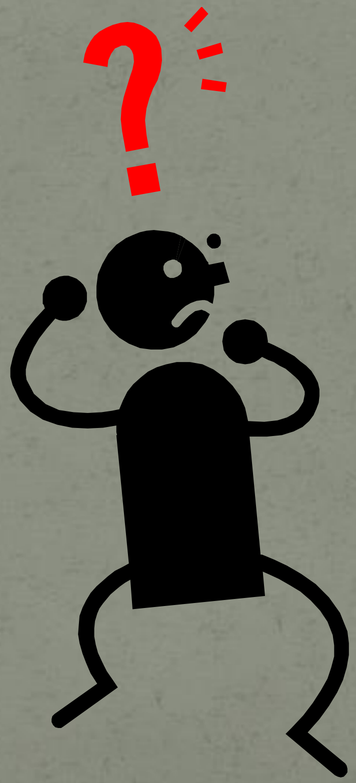


Moment of a force

- For the force shown, we could find its perpendicular distance from the center of rotation
- Or we determine M_O by using the components of F
- The typical sign convention for a moment in 2-D is that counter-clockwise is considered positive. So
- $M_O = (F_Y\, a) - (F_X\, b)$.

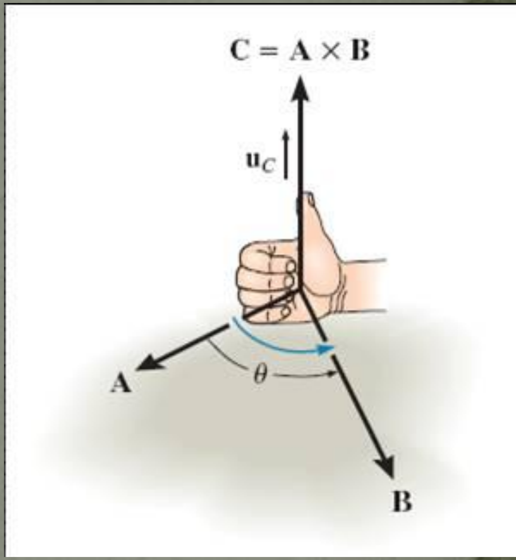






Vector Cross Product

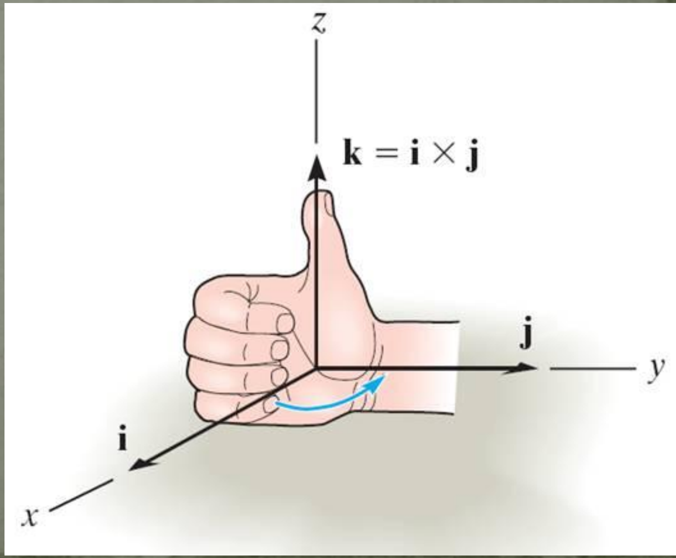
- For some problems e.g. 3-d, finding the relevant perpendicular distance may not be easy
- The vector cross, a vector operation, provides a general method of finding the moment of a force



- $\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}$.
- Magnitude of vector $\boldsymbol{C}$ is A B sin θ
- Direction of $\boldsymbol{C}$ is $\boldsymbol{u_C}$
- where $\boldsymbol{u_C}$ is the unit vector perpendicular to both to the plane containing the vectors A and B vectors

Cross Product: Right Hand Rule

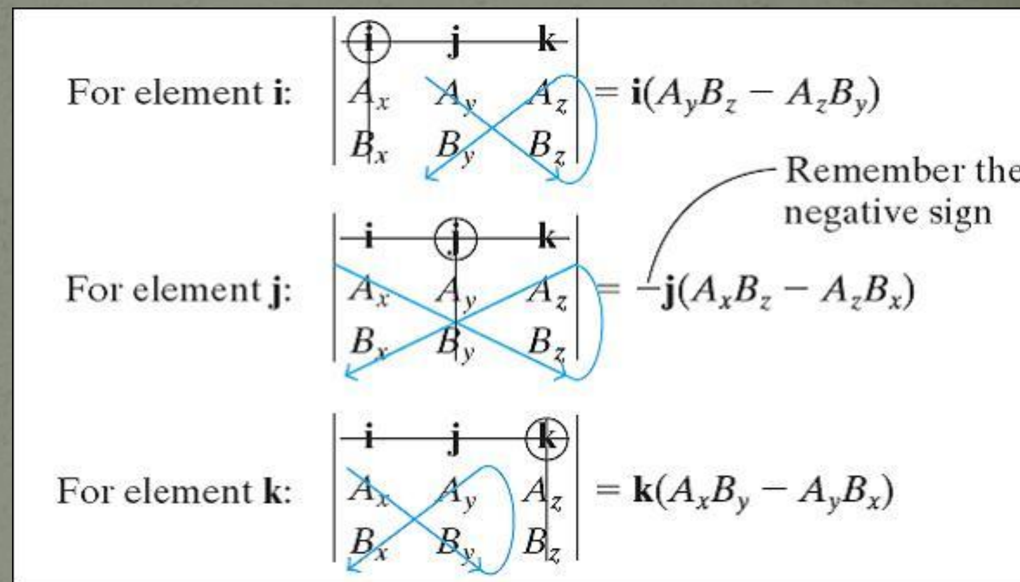
- The right-hand rule is a useful tool for determining the direction of the vector resulting from a cross product
- Note that a vector crossed into itself is zero, e.g.,
  $\boldsymbol{i} \times \boldsymbol{i} = \boldsymbol{0}$
  why?



Cross Product: Determinant

- Alternately we can use the *determinant* of a 2x2 matrix to determine the cross product

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

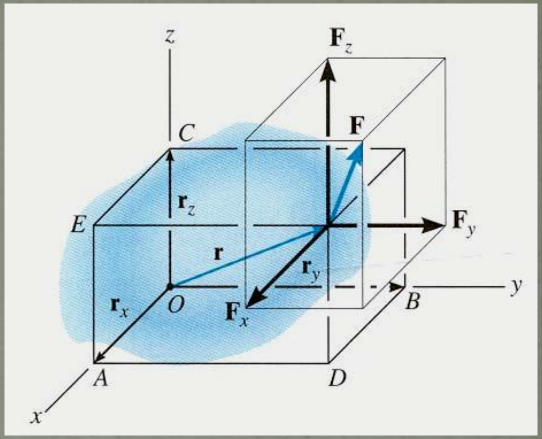


- In fact this is the most common method used

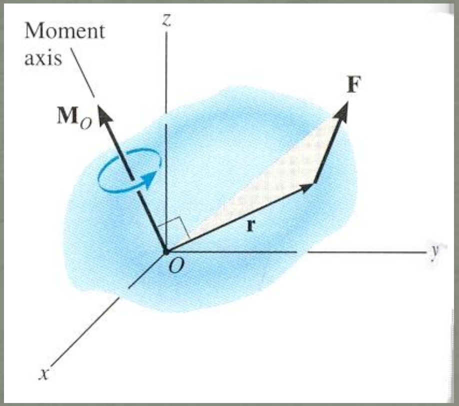
Any Questions So Far ?

Moment of a Force (Vector Formulation)

- Consider the 3-d force vector



- To evaluate the moment we would want to depict it as:



- Clearly, not easy.
- The vector cross product makes it easier

Moment (Vector)

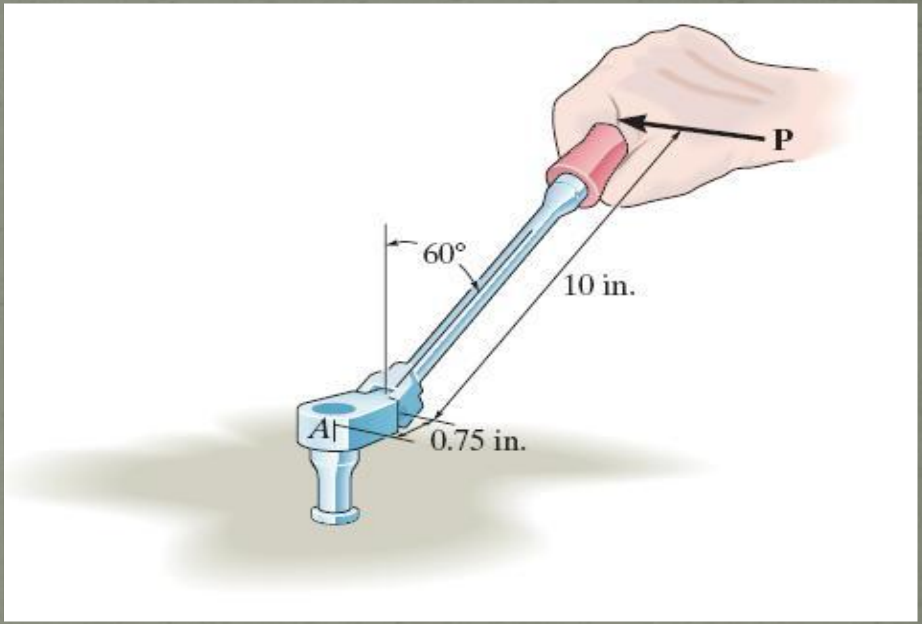
- the vector cross product, $\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$.
- Here $\boldsymbol{r}$ is the position vector *from point O to any point on the line of action of* $\boldsymbol{F}$.
- Using the determinant:

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

- $\boldsymbol{M_O} = (r_y\ F_Z - r_Z\ F_y)\ \boldsymbol{i} - (r_x F_z - r_z F_x)\ \boldsymbol{j} + (r_x F_y - r_y F_x)\ \boldsymbol{k}$

Questions & Comments ?

Moment About an Axis



Scalar Analysis

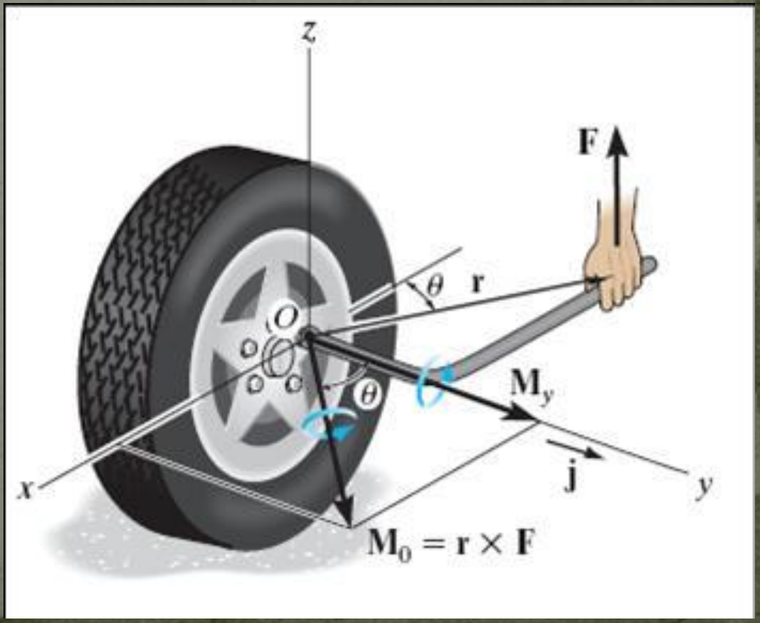
- Recall that the moment of a scalar force about any point O is $M_O = F\, d_O$

where d_O is the perpendicular (or shortest) distance from the point to the *force's line of action*.

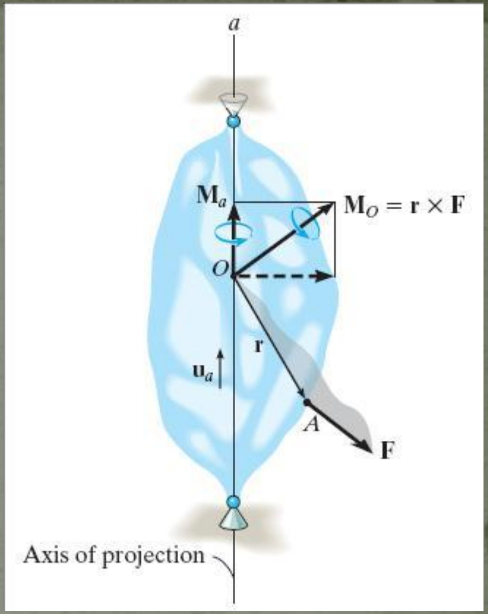
Scalar Analysis

- In this example, the moment about the y-axis $M_y = F_z\,(d_x) = F\,(r\cos\theta)$.

- However, as we saw, such problems are easier if we use vector formulation



Vector Analysis



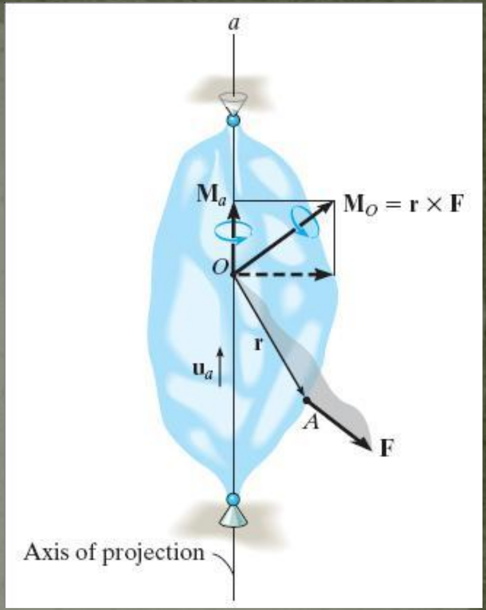
- Let us find the moment of $\boldsymbol{F}$ about the axis a'- a.
- First compute the moment of $\boldsymbol{F}$ about any arbitrary point O that lies on the a'- a axis using the cross product.

$$\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$$

- Next, find the component of $\boldsymbol{M_O}$ along the axis a'- a using the dot product.

$$M_{a'\text{-}a} = \boldsymbol{u_a} \cdot \boldsymbol{M_O}$$

Vector Analysis



Alternately:

- $M_{a'-a}$ can also be obtained by projection as

$$M_a = \mathbf{u}_a \cdot (\mathbf{r} \times \mathbf{F}) = \begin{vmatrix} u_{a_x} & u_{a_y} & u_{a_z} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

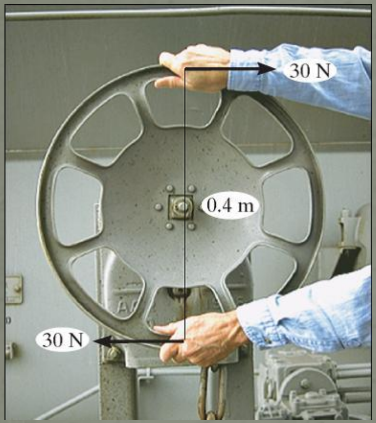
Where $\boldsymbol{u_a}$ represents the unit vector along the axis a'-a axis, $\boldsymbol{r}$ is the position vector from <u>any</u> point on the a'-a axis to any point A on the line of action of the force, and $\boldsymbol{F}$ is the force vector

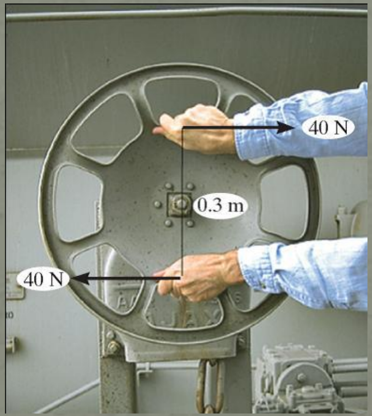
- The above equation is called the triple scalar product

Questions

Moment of a Couple

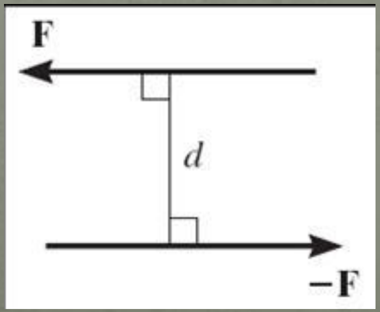
- A couple is defined as two parallel forces with the same magnitude but opposite in direction separated by a *perpendicular distance d*

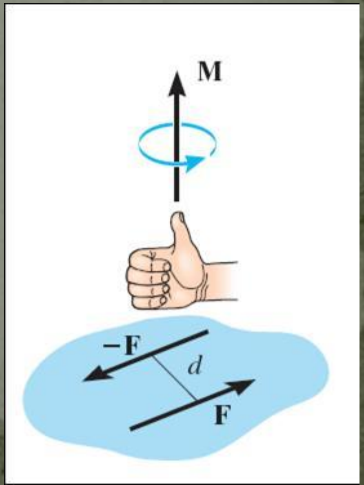




Moment of a Force

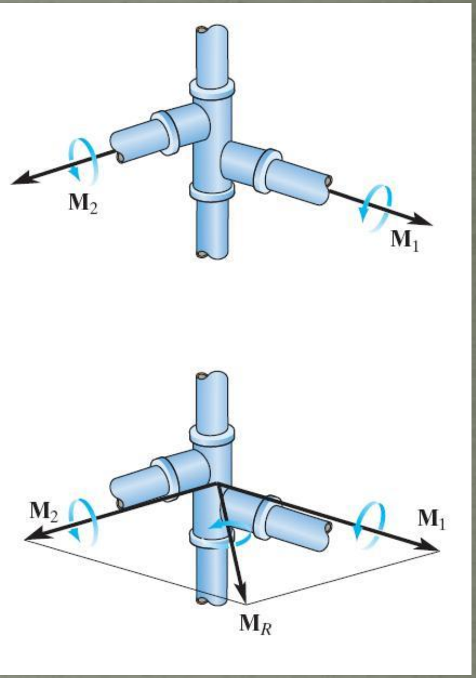
- The moment of a couple is defined as
- $M_O = F\ d$ (using a scalar analysis) or as
- $\boldsymbol{M_O} = \boldsymbol{r} \times \boldsymbol{F}$ (using a vector analysis).
- Here $\boldsymbol{r}$ is any position vector from the line of action of *−F to the line of action of F*.

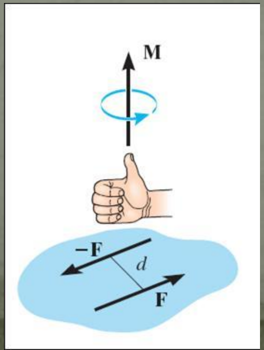




Couples

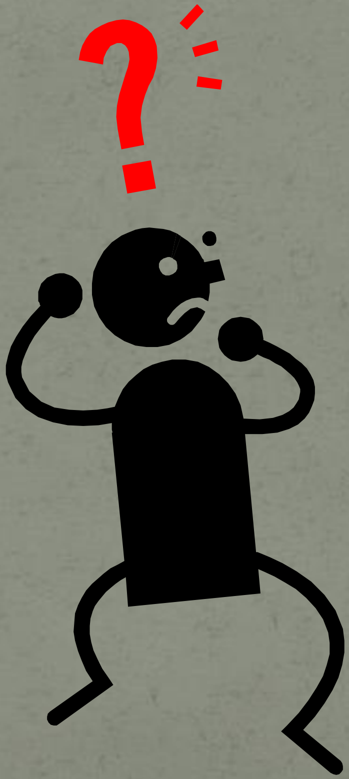
- the net force equals zero and the magnitude of the net moment equals F *d.
- the moment of a couple is a *free vector* because it depends only on the distance between the forces,. It can be moved anywhere on the body and have the same external effect on the body.





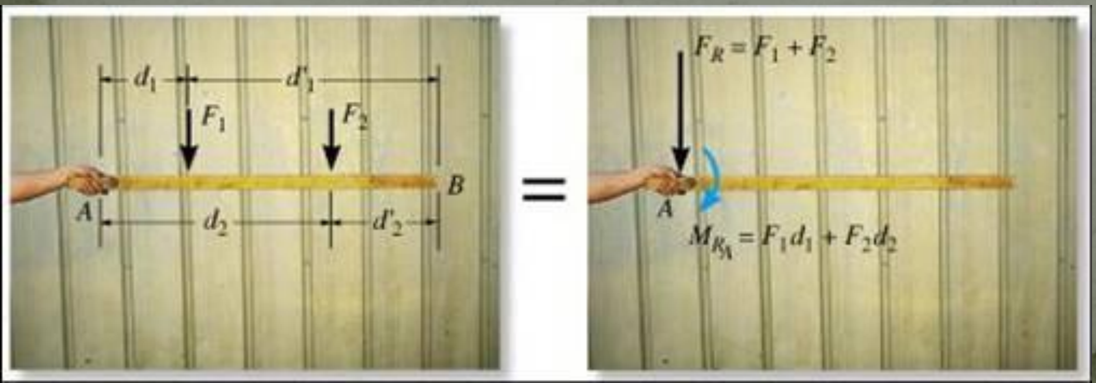
Moments due to couples can be added together using the same rules as adding any vectors.

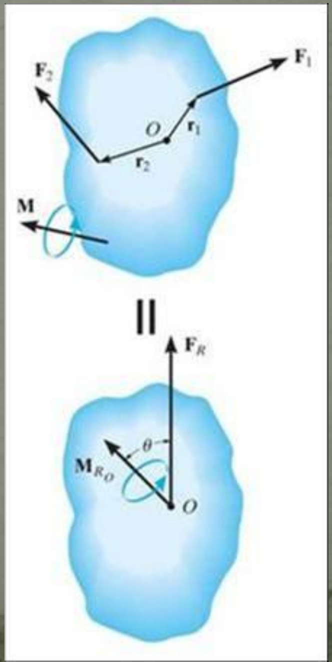
Questions



Simplification of Force & Couple Systems

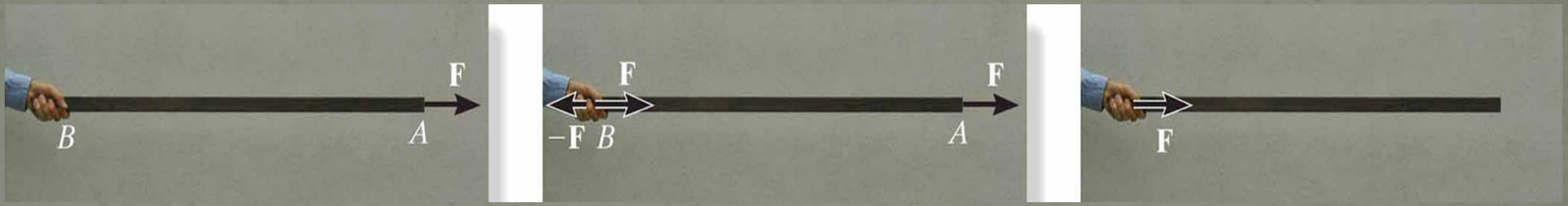
- Consider a number of forces and couple moments are acting on a body
- They can be combined into a single force and couple moment having the same overall effect
- The two force and couple systems are called *equivalent systems*





Moving Force ON Line of Action

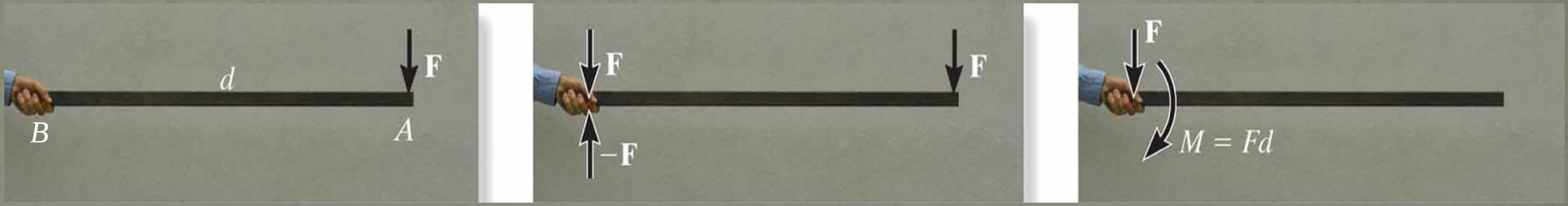
- Moving a force from A to B, when both points are on the vector's line of action, does not change the *external effect*.
- Hence, a force vector is called a *sliding vector*.



- However note that the internal effect of the force on the body does depend on where the force is applied

Moving Force OFF Its Line of Action

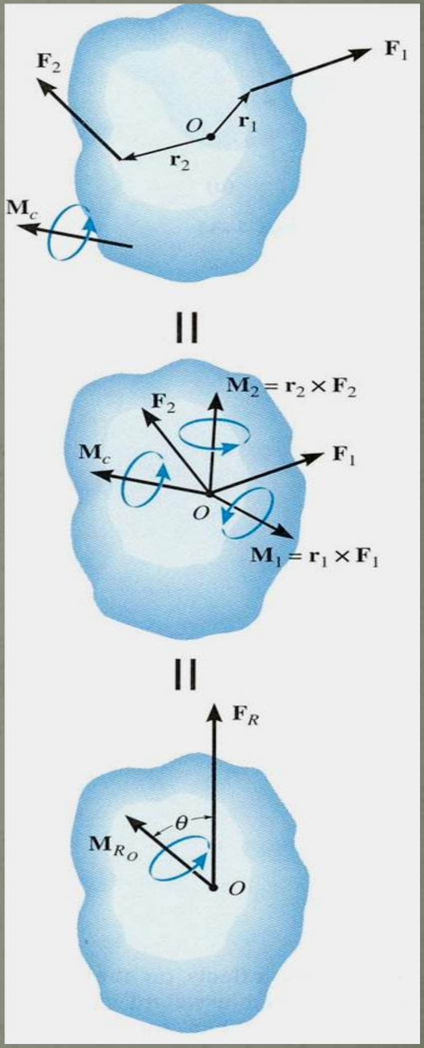
- When a force is moved, but not along its line of action, there is a change in its external effect.
- Moving a force from point A to B requires creating an additional couple moment. So moving a force means you have to “add” a new couple.



- Since this new couple moment is a *“free” vector*, it can be applied at any point on the body.

Simplification of a Force & Couple System

- When several forces and couple moments act on a body, we can move each force and its associated couple moment to a common point O
- We can then add all the forces and couple moments together and find one resultant force-couple moment pair

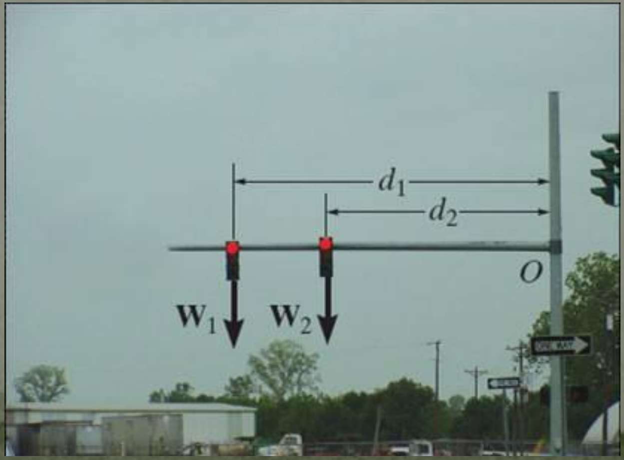


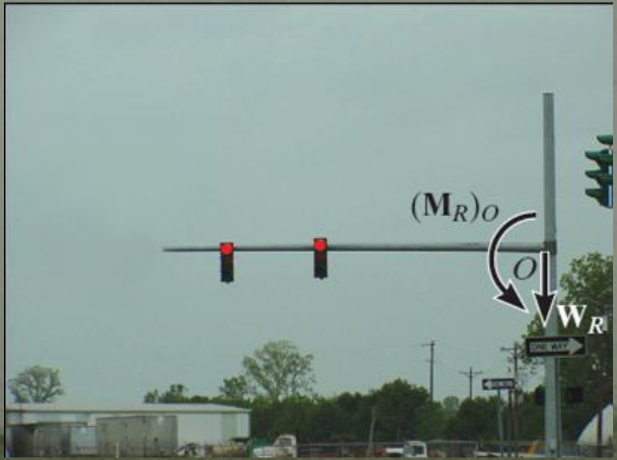
$$\mathbf{F}_R = \Sigma \mathbf{F}$$
$$\mathbf{M}_{R_O} = \Sigma \mathbf{M}_c + \Sigma \mathbf{M}_O$$

Simplification of a Force & Couple System

- If the force system lies in the 2-d x-y plane, then the reduced equivalent system can be obtained using the following three scalar equations.

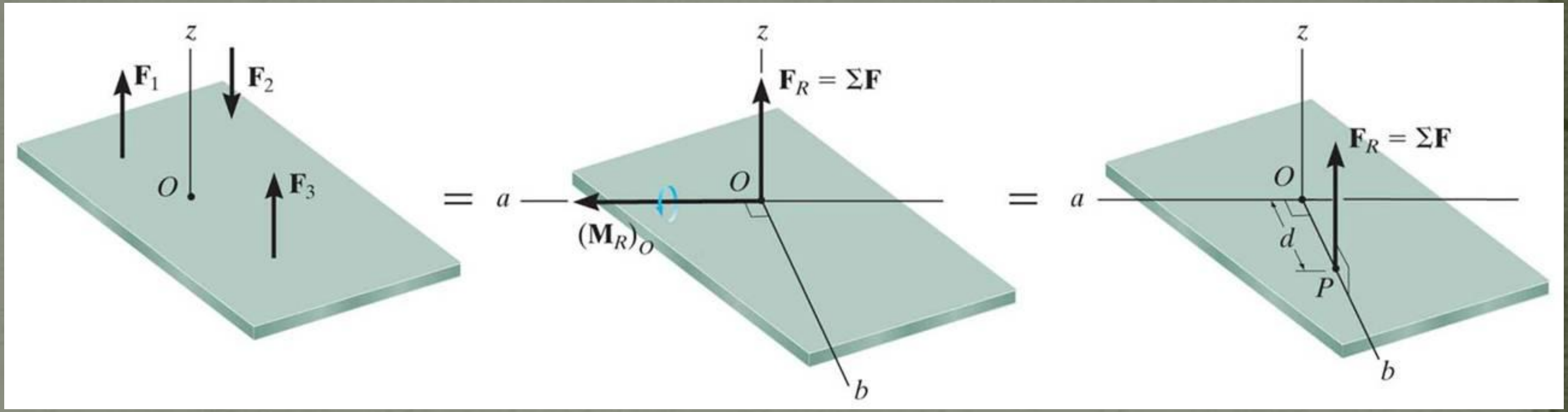
$$F_{R_x} = \Sigma F_x$$
$$F_{R_y} = \Sigma F_y$$
$$M_{R_O} = \Sigma M_c + \Sigma M_O$$





Further Simplification of a Force & Couple System

- If $\boldsymbol{F_R}$ and $\boldsymbol{M_{RO}}$ are perpendicular to each other, then the system can be further reduced to a single force, $\boldsymbol{F_R}$, by simply moving $\boldsymbol{F_R}$ from O to P.

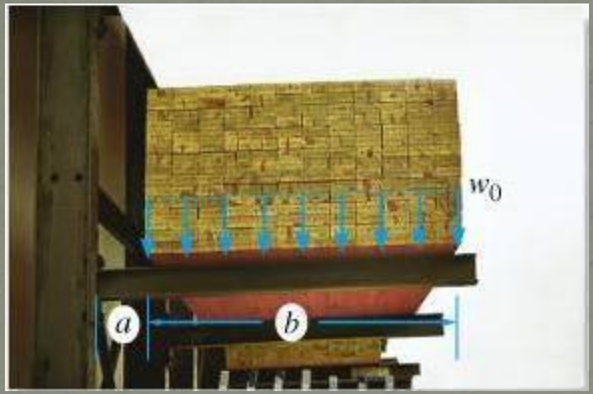


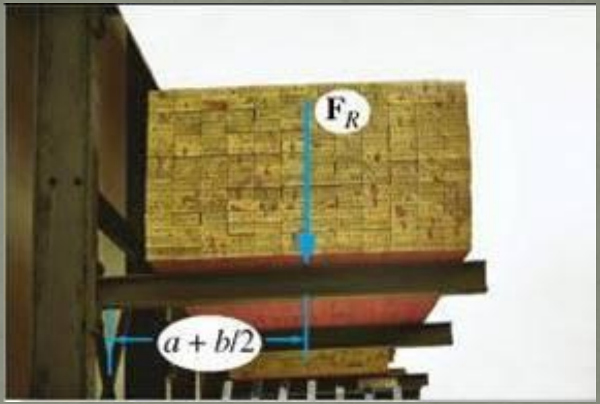
- In three special cases, *concurrent, coplanar, and parallel* systems of forces, the system can always be reduced to a single force.

Any Questions Yet ?

Reduction of a Simple Distributed Loading

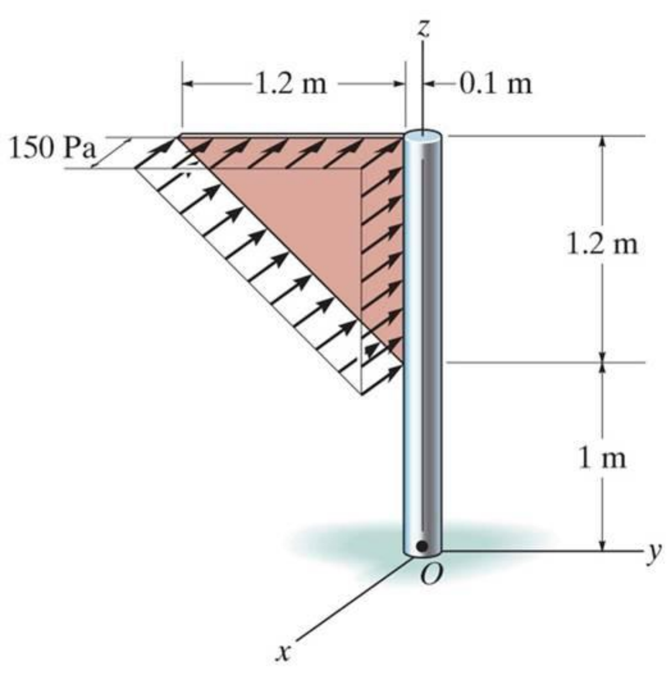
- For some problems, it is often helpful to reduce this distributed load to a single force.





Example

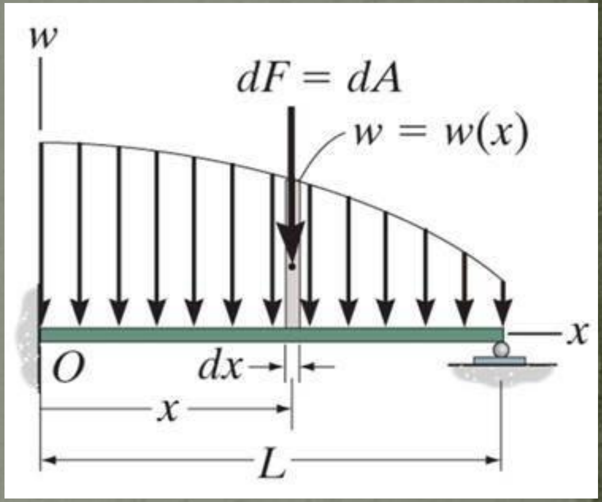
- Consider uniform wind pressure acting on a triangular sign
- To design the joint between the sign and the sign post, we need to determine a single equivalent resultant force and its location



Magnitude of Resultant Force

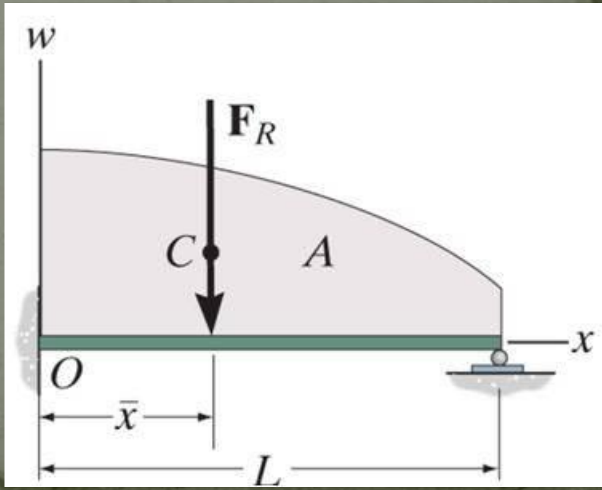
- Consider an element of length dx.
- The force magnitude dF acting on it is given as

$$dF = w(x)\,dx$$



- The *net force* on the beam is given by

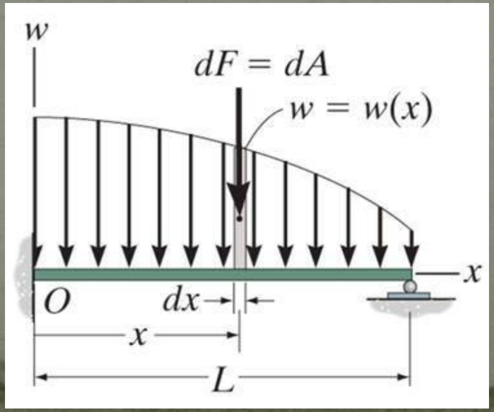
$$+\downarrow \; F_R = \int_L dF = \int_L w(x)\,dx = A$$

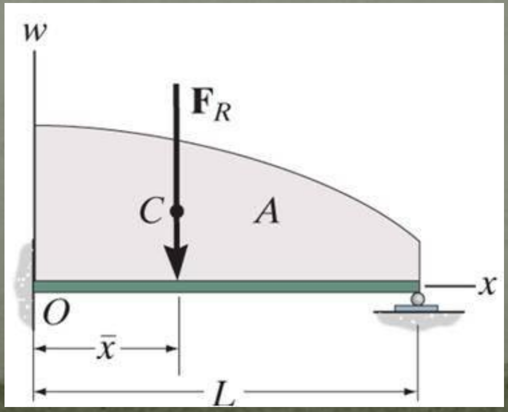


where A is the area under the loading curve w(x)

Location of the Resultant

- The force dF will produce a moment of (x)(dF) about point O.
- The total moment about point O is given as
  $\curvearrowleft + \; M_{RO} = \int_L x \, dF = \int_L x \, w(x) \, dx$
- Assuming that F_R acts at, it will produce the moment about point O as
  $\curvearrowleft + \; M_{RO} = (\bar{x})(F_R) = \bar{x} \int_L w(x) \, dx$

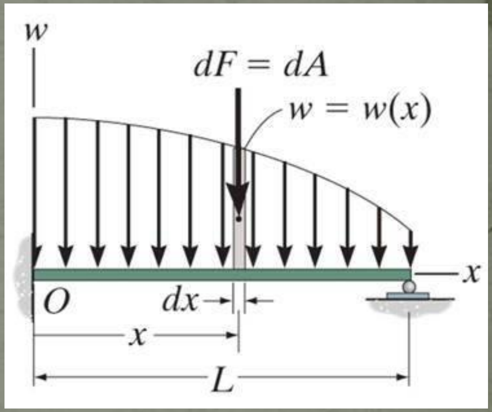


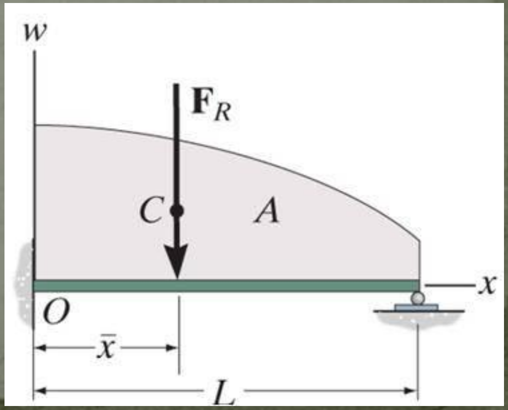


Location of the Resultant

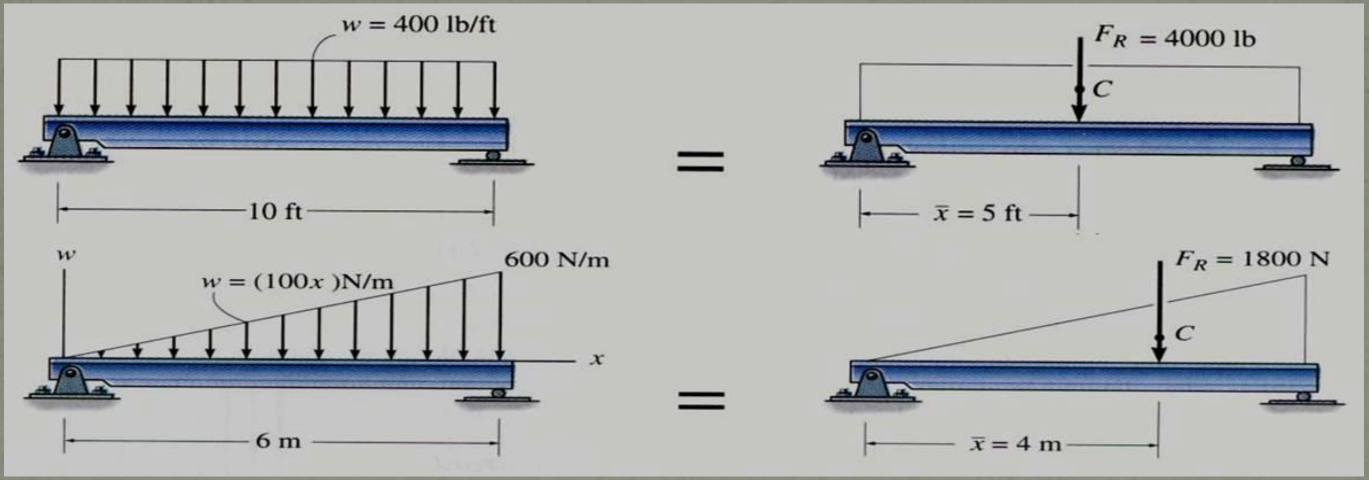
- Comparing the last two equations we can find the location of the point of action of the resultant of the distributed load

$$\bar{x} = \frac{\int_L x w(x)\, dx}{\int_L w(x)\, dx} = \frac{\int_A x\, dA}{\int_A dA}$$



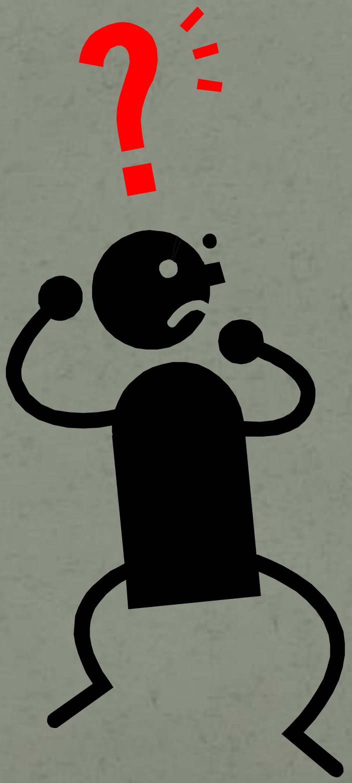


Examples



- What do you notice about the location of the equivalent of the rectangular and triangular loadings ?

Questions & Comment



- Now we solve problems