Internal Forces

Chapter 7

Overview

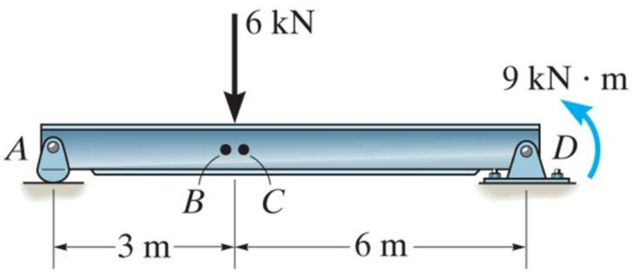
- Internal Forces
- Shear Force & Bending Moment Diagrams
- Cables

Internal Forces

- The design of any structural member requires finding the forces acting within the member to make sure the material can resist those loads

Internal Forces - Analysis

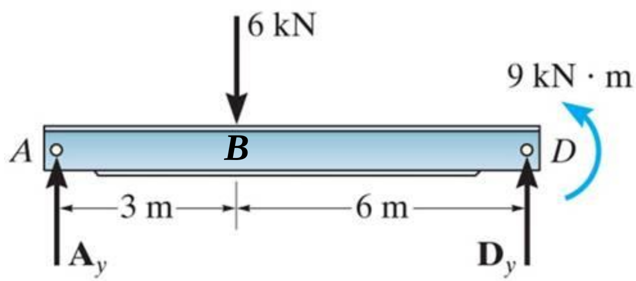
- Consider the beam subject to the applied loads



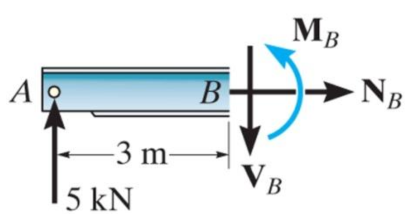
- The structural engineer wants to know the internal force at B in the member

Internal Forces - Analysis

- Step 1: determine the support reactions



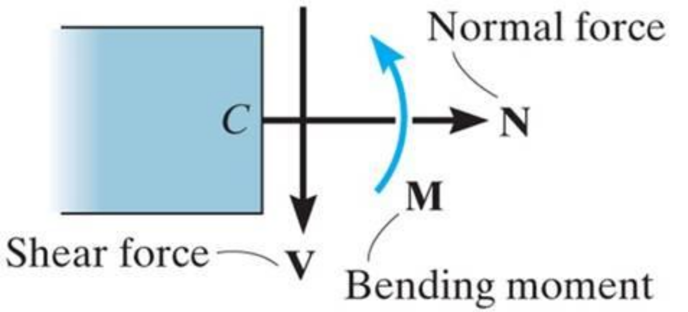
- Step 2: cut the beam at B and draw a FBD of one of the halves of the beam (Method of Sections)



- Step 3: solve for the unknowns using the equations of equilibrium

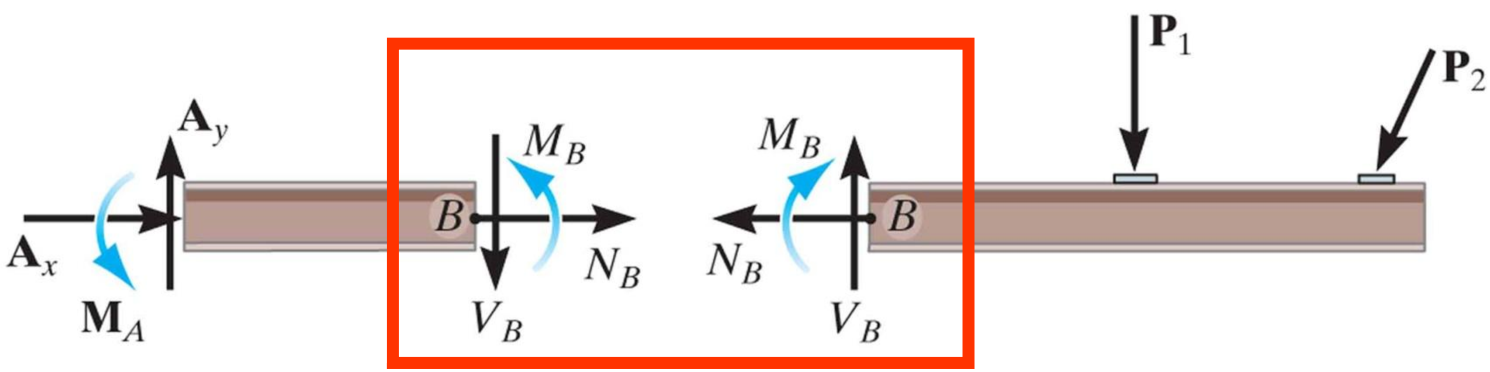
Types of Internal Forces

- In two-dimensional cases, typical internal loads are
  - *normal* or *axial forces* *e.g.* tension, compression (N, acting perpendicular to the cross section),
  - *shear forces* (V, acting along the section),
  - *bending moment* (M).
  - *Torsional moment* or *twisting* about the longitudinal axis of the member
- Internal forces are the basis of the design of many structural members/ systems



Types of Internal Forces

- For a member, the loads on the left and right sides of the section at B *are equal in magnitude but opposite* in direction.
- This is because when the two sides are reconnected, the net loads are zero at the section



Questions ?

Shear Force & Bending Moment Diagrams

- During the design of a member, e.g. a beam, a structural engineer needs to know the shear force or bending moment at all points along the member.
- A graphical display of this analysis is called a shear force diagram and bending moment diagram respectively
- The diagram enables the engineer to see where the critical shear or bending moment will occur, its value, and hence design the member accordingly

Shear Force Diagram

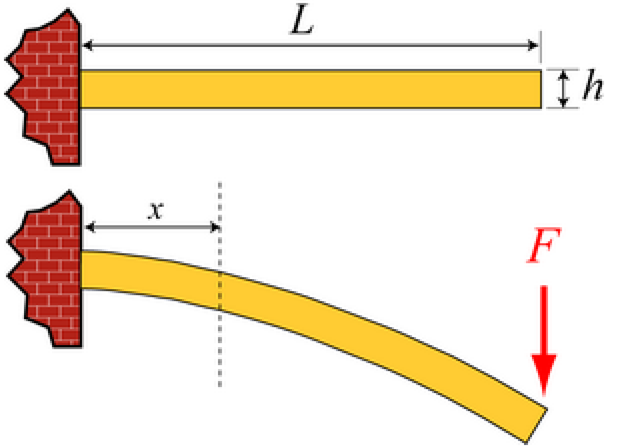
- By definition, the shear force at any point equals the sum of the loads and reactions from a reaction or end of member to that point.
- Loads and reactions acting upwards are positive
- The shear diagram is straight and sloping over uniformly distributed loads
- The shear diagram is straight and horizontal between concentrated loads
- The shear is a vertical line and undefined at points of concentrated loads.

Bending Moment Diagram

- By definition, the bending momemt at any point equals the sum of the moments and couples from a reaction or end of member to that point.
- Clockwise moments about the point are positive
- The maximum moment occurs where the shear force is zero
- The bending moment diagram is straight and sloping between concentrated loads
- The bending moment diagram is curved over uniformly distributed loads.

Force Versus Stress

- Normal forces include *tensile forces* and *compressive forces*
- Tensile force/cross-section area, is called *tensile stress* aka *pressure*. Ditto compressive stress
- Shear force/cross-section area, is called *shear stress*
- The stress associated with bending moment is called *bending stress* or *flexural stress*. Ditto torsional moment



Relations Between Distributed Load, Shear & Moment

It can be shown that (see text for proofs):

1. Slope of shear diagram = distributed load intensity

$$\boxed{\frac{dV}{dx} = w(x)}$$

2. Change in shear = area under loading curve

$$\boxed{\Delta V = \int w(x)dx}$$

Relations Between Distributed Load, Shear & Moment

3. Slope of moment diagram = shear

$$\boxed{\frac{dM}{dx} = V}$$

4. Change in moment = area under shear force diagram

$$\boxed{\Delta M = \int V dx}$$

Influence Lines

- This topic is not in your textbook. However it is very relevant for “real-life” analyses
- For example; drawing the bending moment diagram for a bridge member as a truck passes over it.
- I will provide material on this for students to use on their projects, if requested.

Questions ?

- Let’s work examples, several !

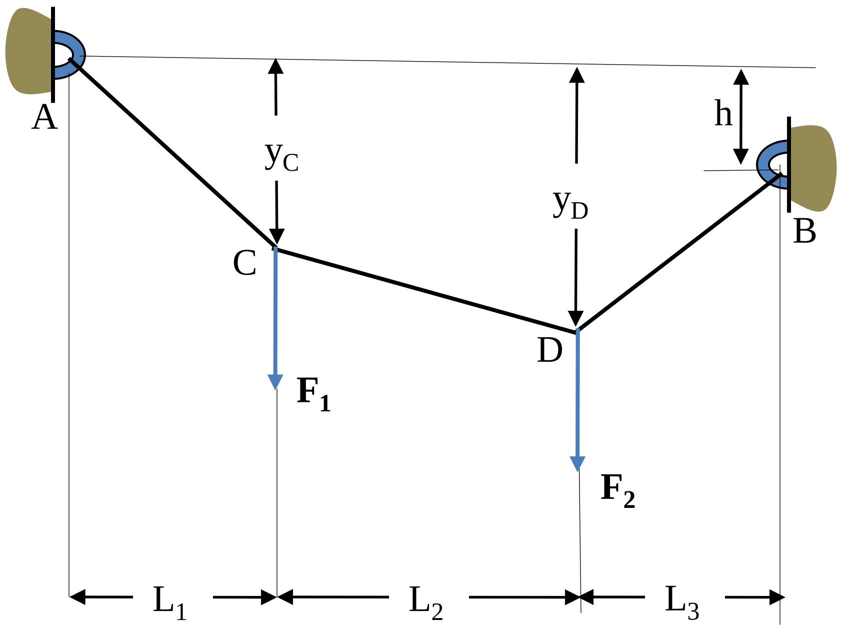
Cables

- Flexible cables and chains are often used in structures for support or to transmit loads from one member to another
- An *ideal cable* is assumed to be completely flexible, massless, inextensible

Cable Carrying a Concentrated Loads

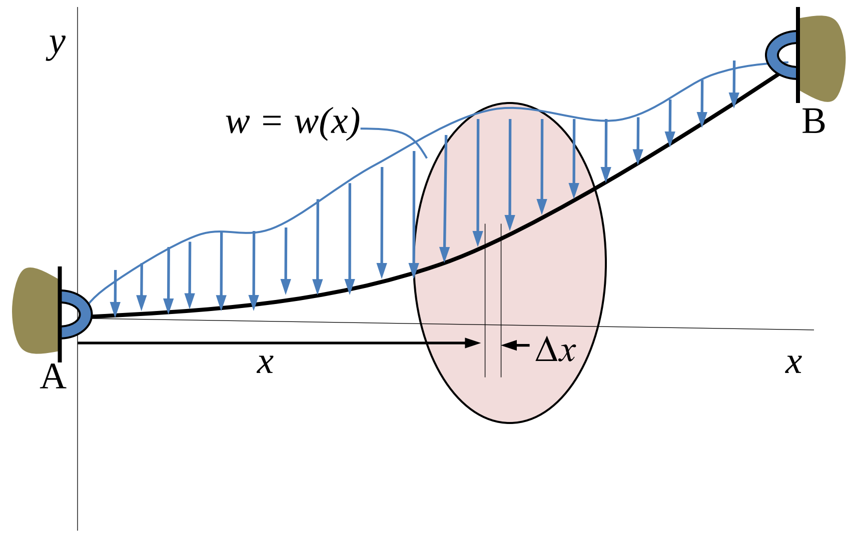
- Generally, the Method of Joints and the Method of Sections can be applied to analyze the cable.

- However there are some special cases.



Cable Carrying a Distributed Load

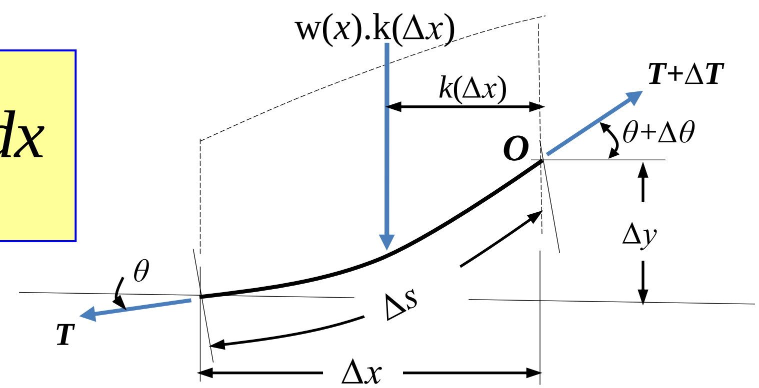
- Equations of Equilibrium: (see text for full proofs)



$$\sum F_x = 0, \quad \boxed{T\cos\theta = const = F_H}$$

$$\sum F_y = 0, \quad \boxed{T\sin\theta = \int w(x)dx}$$

$$\sum M_O = 0, \quad \boxed{\tan\theta = \frac{dy}{dx} = \frac{1}{F_H}\int w(x)dx}$$



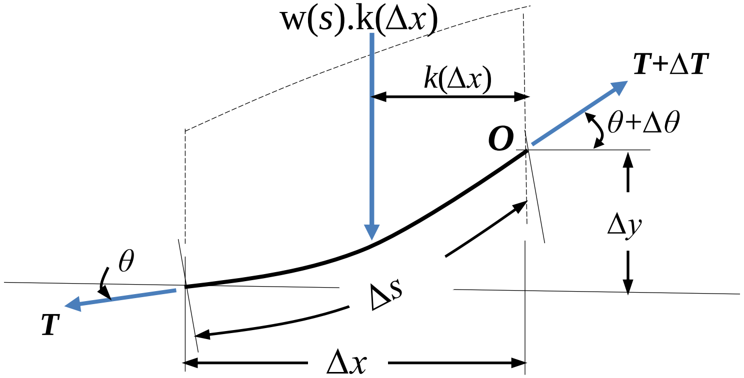
$$\boxed{Also,\ y = \frac{1}{F_H}\int\left(\int w(x)dx\right)dx}$$

Cable Subject to Its Self Weight

- In some cases the self weight of the cable is relevant to the analysis (e.g. electrical transmission line)
- Since the self weight is distributed uniformly along the length of the cable, the cable will have the shape of a *catenary*

- It can be shown that (see proof in text)

$$x = \int \frac{ds}{\left[1 + \frac{1}{F_H^2}\left(\int w(s)\,ds\right)^2\right]^{\frac{1}{2}}}$$



Questions & Comments ?

- More problem solving.
- Now !!