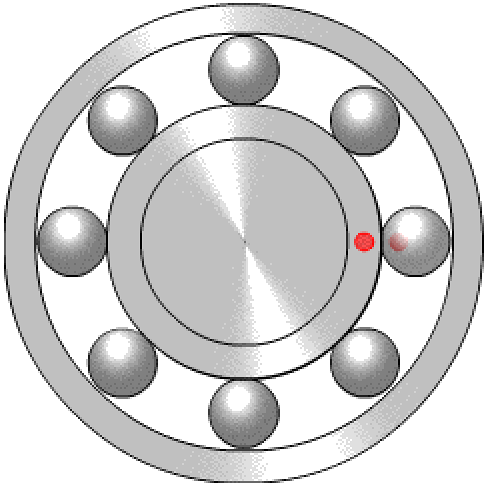
Friction

Chapter 8

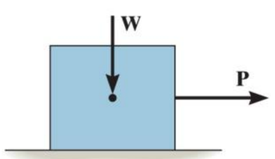
Overview

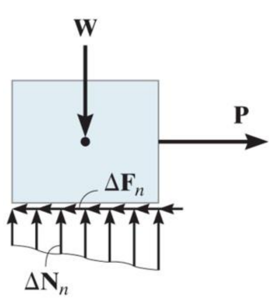
- Dry Friction
- Wedges
- Flatbelts
- Screws
- Bearings
- Rolling Resistance

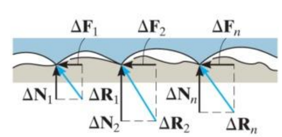


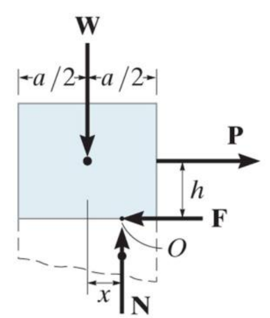
Dry Friction

- Friction is defined as a force of resistance acting on a body which prevents slipping of the body relative to a second body
- Empirical evidence shows that frictional forces act tangent to the contacting surface in a direction opposing the relative motion or tendency for motion.
- Equilibrium: F = P,   N = W, and W*x = P*h.





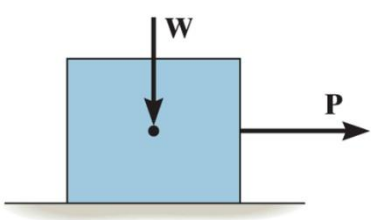


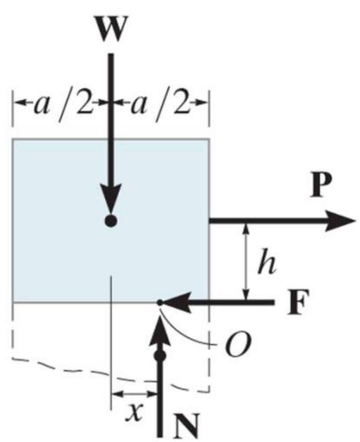


Resultant Normal
and Frictional Forces

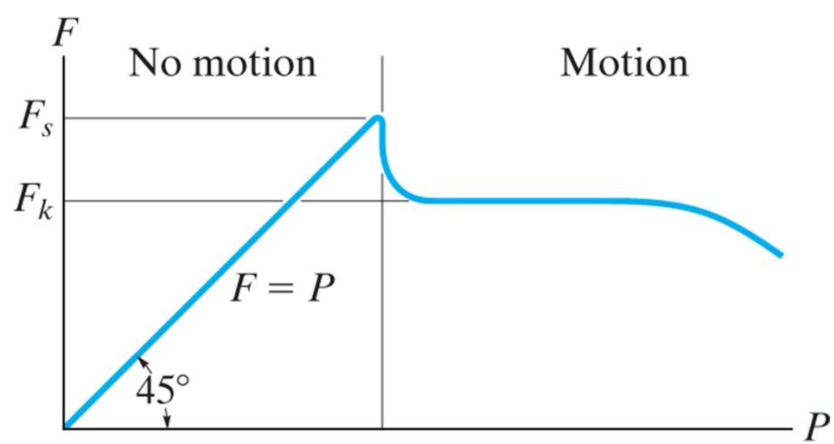
Dry Friction

- Assume that tipping does not occur (i.e., “h” is small or “a” is large).
- As we gradually increase the magnitude of the force P. the friction force F varies with P,



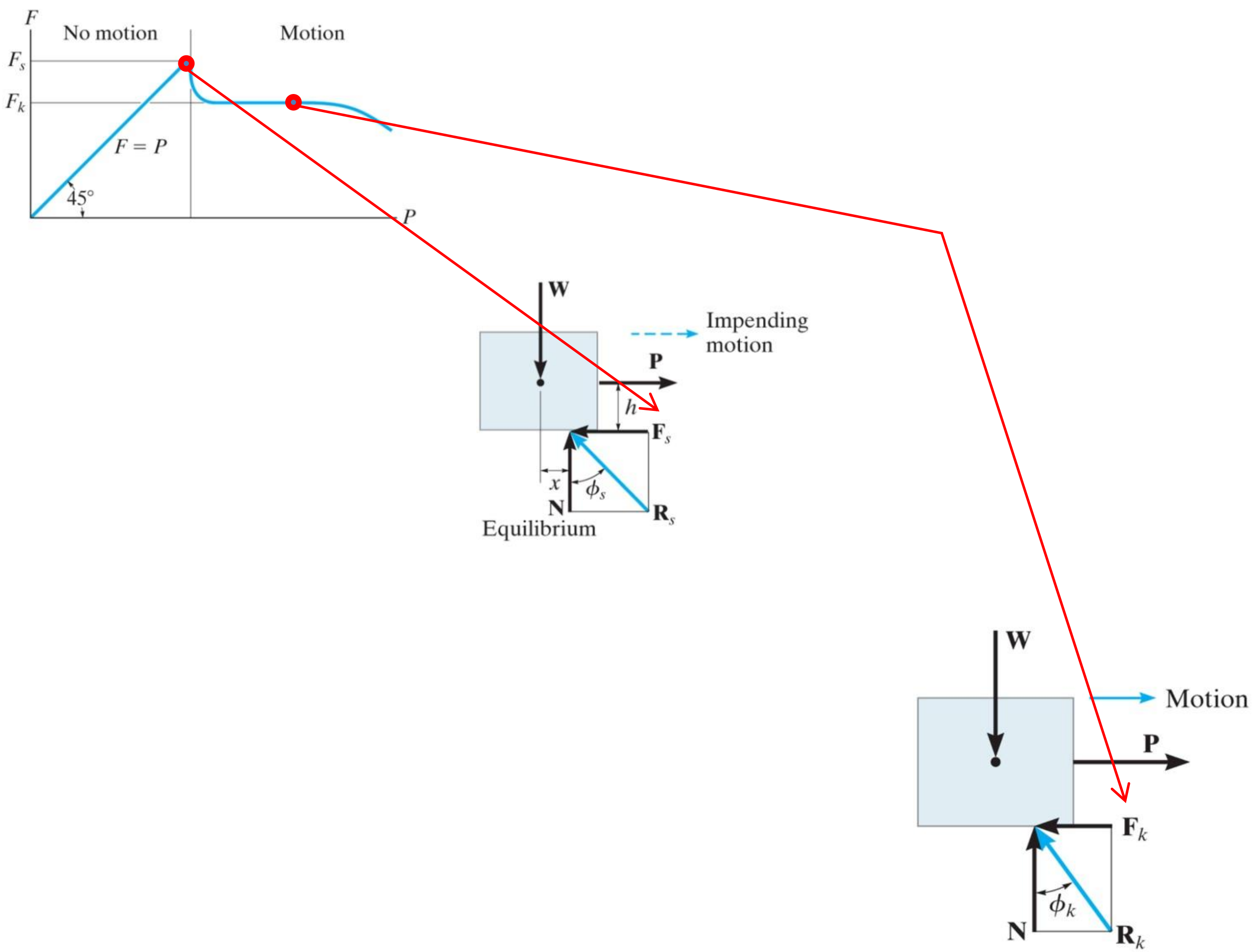


Resultant Normal and Frictional Forces



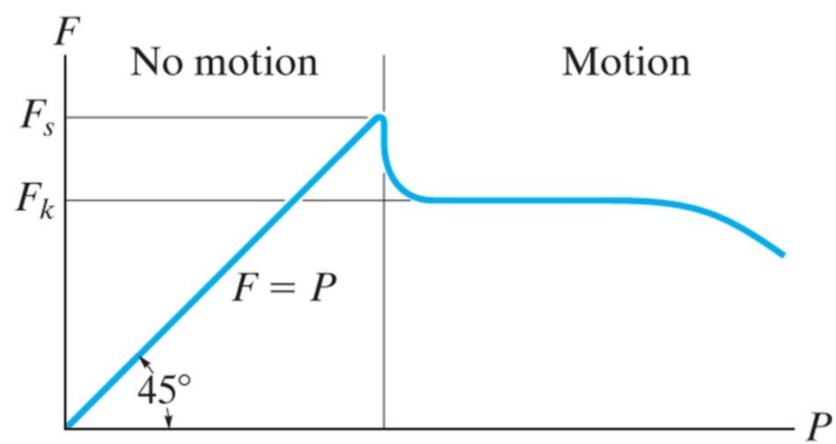
Dry Friction

- The *maximum friction force* is attained just before the block begins to move (a situation that is called "impending motion").
- The value of the force is found using $F_s = \mu_s N$, where μ_s is called the coefficient of static friction. μ_s depends on the two materials in contact.
- Once the block begins to move, the frictional force typically drops and is given by $F_k = \mu_k N$. The value of μ_k is the coefficient of kinetic friction and is less than μ_s.


F
No motion
Motion
F_s
F_k
F = P
45°
P
W
Impending motion
P
h
F_s
x
φ_s
N
R_s
Equilibrium
W
Motion
P
F_k
φ_k
N
R_k

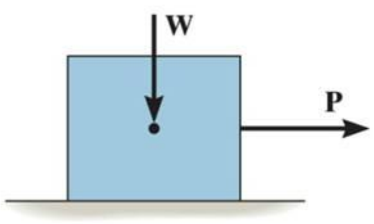
Dry Friction

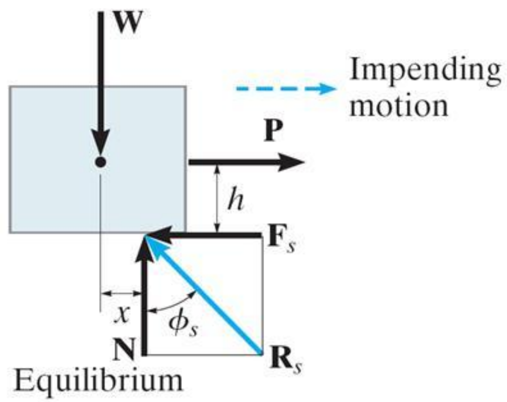
- It is important to note that the friction force may be less than the maximum friction force.
- If the object is not moving, don't assume the friction force is at its max. value of $F_s = \mu_s N$ unless you are told or know motion is impending



Determing μ_s

- on the verge of sliding, the block just begins to slip, the maximum friction force is $F_s = \mu_s N$
- Thus, N and F_s combine to create a resultant R_s



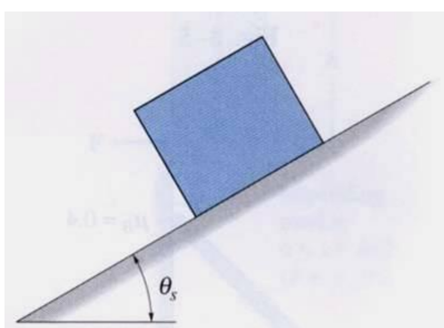


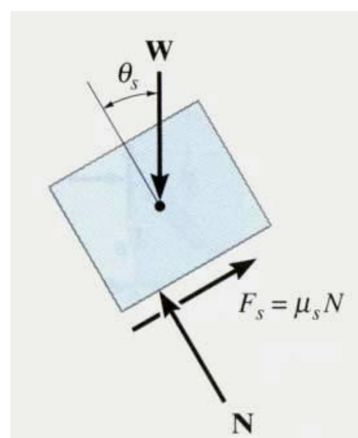
- From the figure,

  $\tan \phi_s = (F_s / N) = (\mu_s N / N) = \mu_s$

Determing μ_S - Inclined Plane

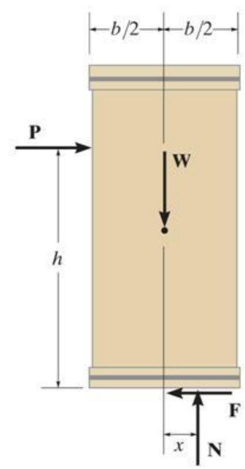
- Analysis of the block just before it begins to move gives (using $F_s = \mu_s N$):
- $\nwarrow + \ \sum F_y = N - W\cos\theta_s = 0$
- $\nearrow + \ \sum F_X = \mu_S N - W\sin\theta_s = 0$
- $\mu_s = (W\sin\theta_s)/(W\cos\theta_s) = \tan\theta_s$

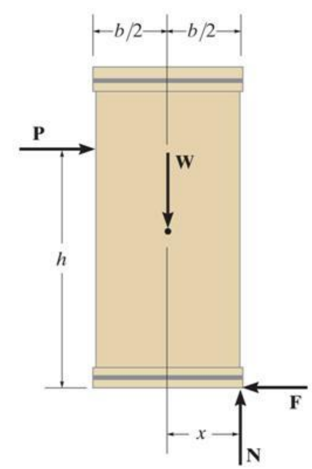




Impending Tipping Versus Slipping

- how can we determine if the block will slide or tip first?
- In this case, we have four unknowns (F, N, x, and P) and only three E-of-E.
- we have to make an assumption to give us another equation (the friction equation!). Then we can solve for the unknowns
- Finally, we need to check if our assumption was correct.





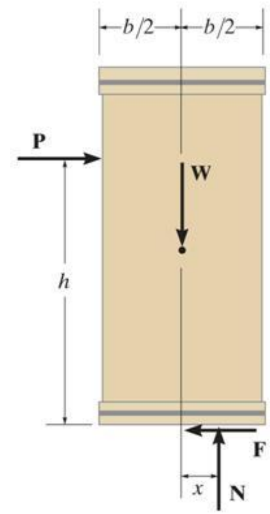
Impending Tipping Versus Slipping

- Assumption: Slipping occurs

  Known: $F = \mu_s N$

  Solve for: x, P, and N

  Check: $0 \leq x \leq b/2$

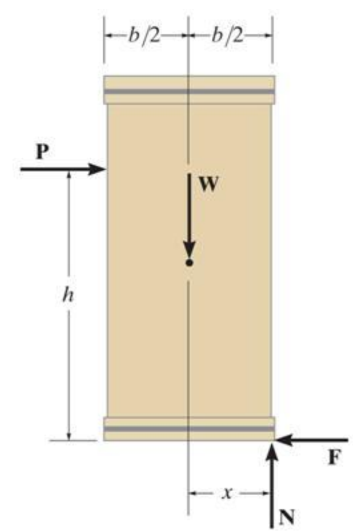


- Assumption: Tipping occurs

  Known: $x = b/2$

  Solve for: P, N, and F

  Check: $F \leq \mu_s N$



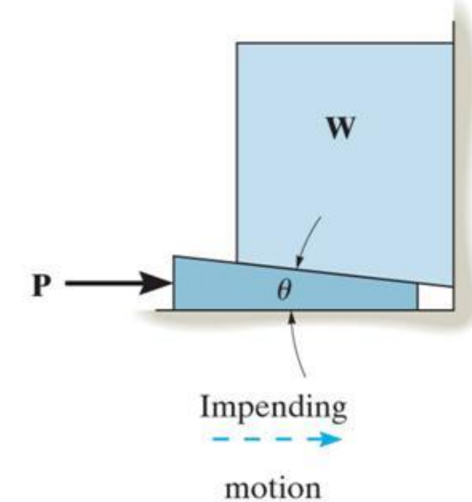
WEDGES AND FRICTIONAL FORCES ON FLAT BELTS

- Wedges are used to adjust the elevation or provide stability for heavy objects

- Belt drives are commonly used for transmitting the torque developed by a motor to a wheel attached to a pump, fan or blower.

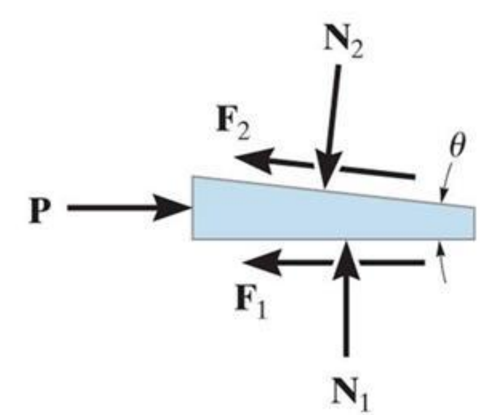
Analysis of a Wedge

- A wedge is a *simple machine* in which a small force P is used to lift a large weight W



- First we draw the free body diagram of the wedge, noting that

1) the friction forces are always in the *direction opposite to the motion*,
2) the friction forces are along the contacting surfaces; and,
3) the normal forces are perpendicular to the contacting surfaces



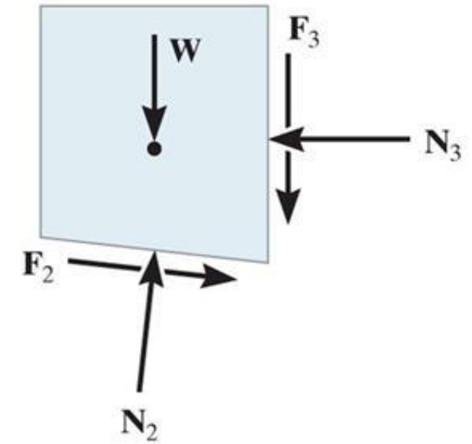
Analysis of a Wedge

- Next, we look at the object on top of the wedge noting

1) a*t the contacting surfaces* between the wedge and the object *the forces are equal in magnitude* and opposite in direction to those on the wedge; and,

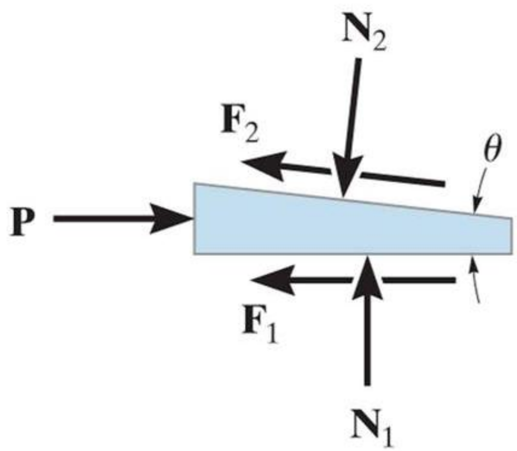
2) all other forces acting on the object should be shown.

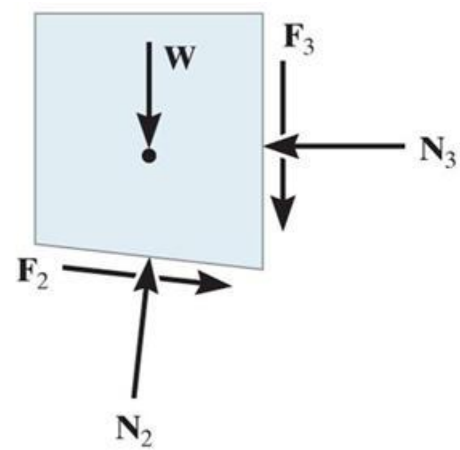
3) $\sum F_x = 0$ and
$\sum F_y = 0$, for the wedge and the object
Also, for *the impending motion frictional equation*, $F = \mu_S N$.



Analysis of a Wedge

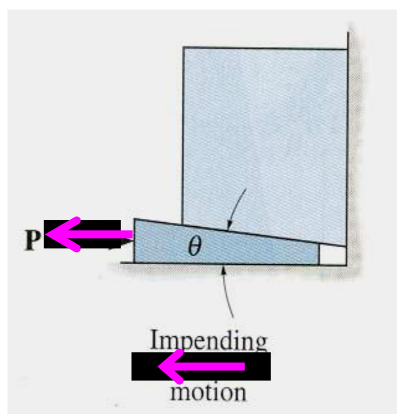
- Start by analyzing the free body diagram in which the number of unknowns are less than or equal to the number of equations of equilibrium and frictional equations





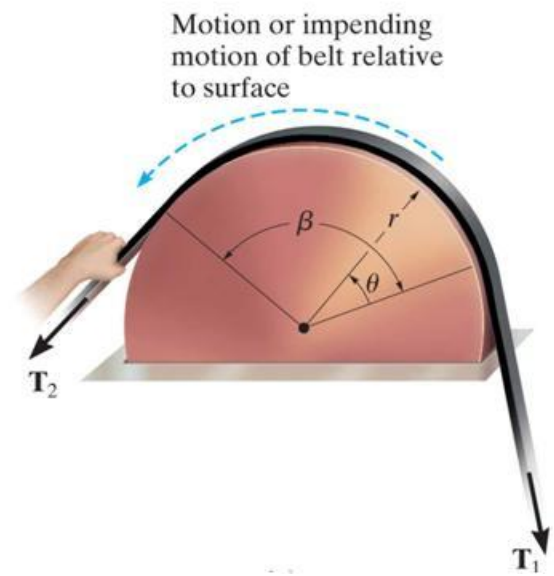
Analysis of a Wedge

- If the object is to be lowered, then the wedge needs to be pulled out.
- If the value of the force P needed to remove the wedge is positive, then the wedge is *self-locking*, i.e., it will not come out on its own.



Analysis of Flat Belt

- Consider a flat belt passing over a fixed curved surface with the total angle of contact equal to β (in radians)
- If the belt slips or is just about to slip, then T_2 must be larger than T_1 plus the motion resisting friction forces. Hence, T_2 must be greater than T_1.



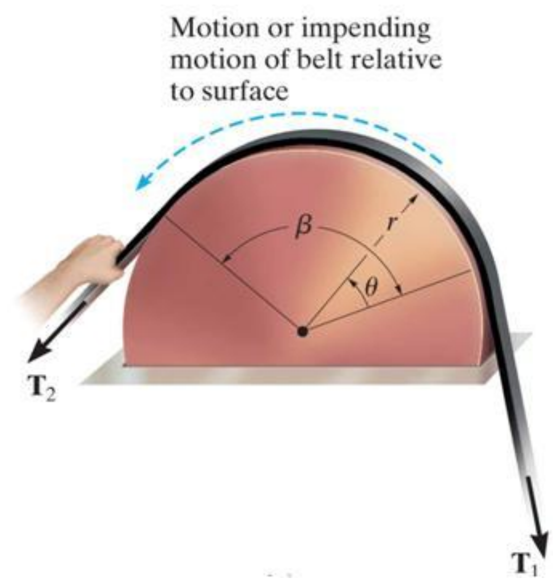
Analysis of Flat Belt

- It can be shown that

$$T_2 = T_1 \, e^{\mu \beta}$$

where μ is the coefficient of static friction between the belt and the surface. [see text for proof]

- Remember to use radians when using this formula!!
-



Analysis of Screws

- Screws are used as fasteners or to transmit power or motion from one machine part to another
- Screws can be classified by the *thread*. E.g. *square-threaded screw, V-thread*
- A screw is considered a cylinder called a *barrel* or *shaft*, with the *thread* wrapped around it.

fig08_17a.jpg

Analysis of Screws

- If we unwind the thread by one revolution, the slope or *lead angle* is given by

$$\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right)$$

- *l* is called the *lead* of the screw, and is the distance advanced by turning the screw one revolution

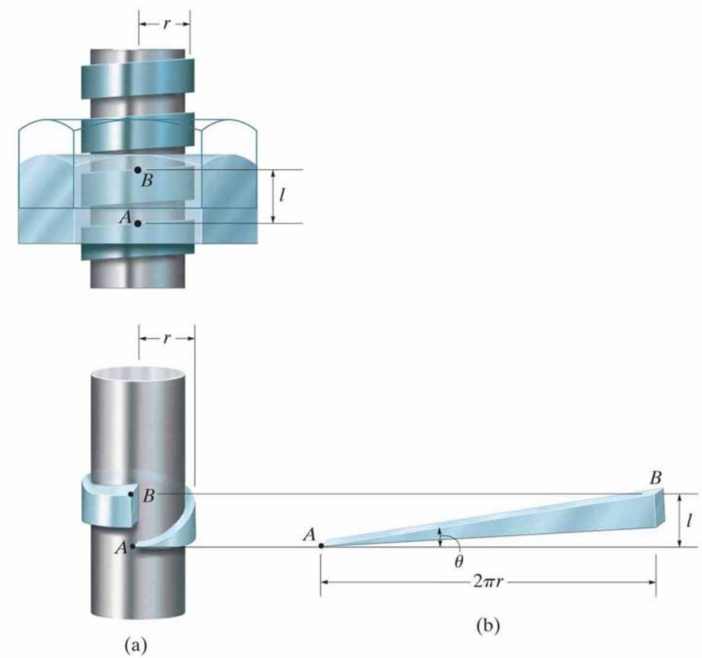


fig08_14.jpg

Upward Impending Motion

- Consider a square-threaded screw subject to impending motion due to an applied torque **M**.
- The free body diagram of the entire unraveled thread through can be represented as follows
  where **W** is the vertical force on the or the axial force on the shaft, and
  R is the reaction of the groove on the thread. **R** has frictional and normal components

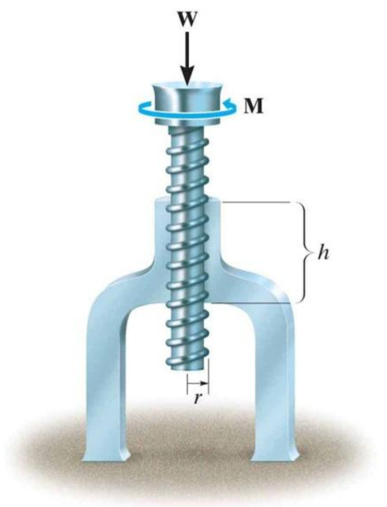
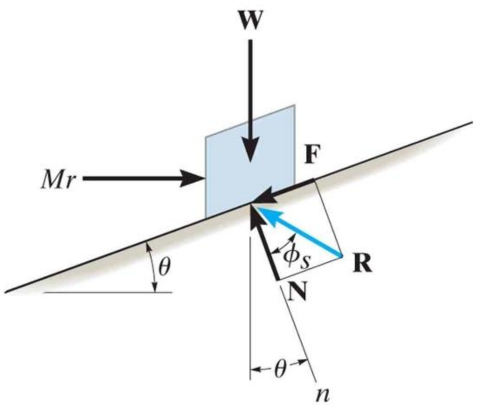


fig08_15.jpg



Upward screw motion
(a)

fig08_16a.jpg

Upward Impending Motion

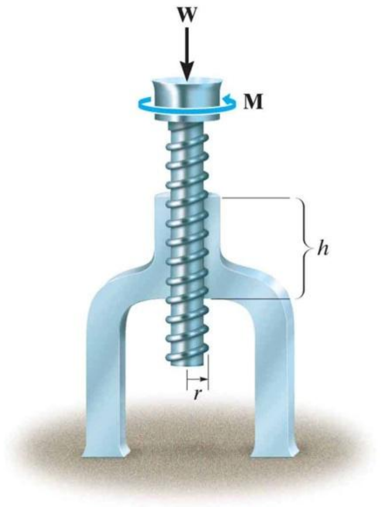


fig08_15.jpg

- The horizontal force associated with the couple moment *M* is *M/r*.
- The frictional component $F = \mu_s N$
- The angle of static friction

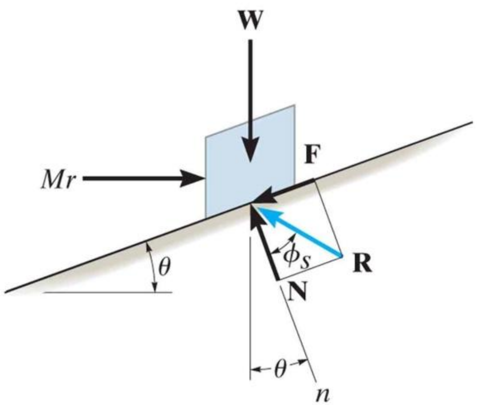
$$\phi_s = \tan^{-1}\left(\frac{F}{N}\right) = \tan^{-1}\mu_s$$

- By Equations of Equilibrium

$$\rightarrow \sum F_x = 0;\ {M}/{r} - R\sin(\phi_s - \theta) = 0$$

$$\uparrow \sum F_y = 0;\ R\cos(\phi_s + \theta) - W = 0$$

$$\boxed{M = rW\tan(\phi_s + \theta)}$$



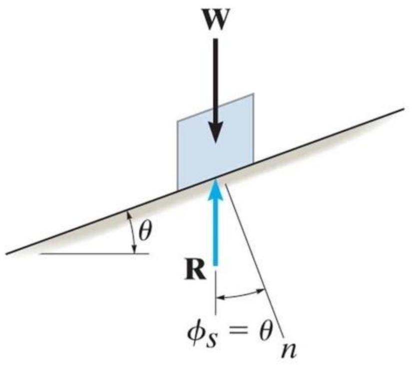
Upward screw motion

(a)

fig08_16a.jpg

Self-Locking Screw

- A screw is *self-locking* if it remains in place under any axial load **W** when the moment **M** is removed.
- In this case **R** acts on the other side of **N**.
- If , $\phi_s = \theta$ then **R** will act vertically to balance **W**, and the screw will be on the verge of winding downwards



Self-locking screw ($\theta = \phi_S$)
(on the verge of rotating downward)

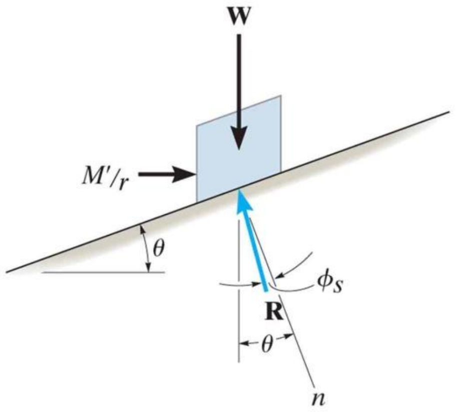
(b)

fig08_16b.jpg

Downward Impending Motion

- If a screw is self-locking, a couple **M**’ must be applied in the opposite direction to wind the screw downward
- $\phi_s > \theta$
- This causes a horizontal force in the reverse direction that will push the thread downwards
- Using the previous procedure it can be shown that

$$\boxed{M' = rW\tan(\theta - \phi_s)}$$



Downward screw motion ($\theta > \phi_S$)

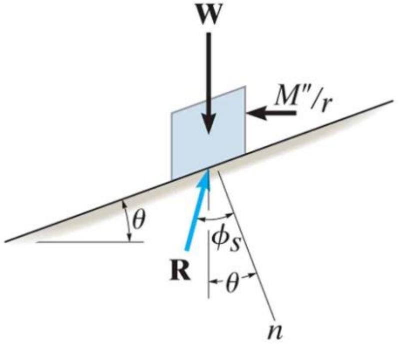
(c)

fig08_16c.jpg

Downward Impending Motion

- If the screw is not self-locking it is necessary to apply a moment M” to prevent the screw from winding downwards
- $\phi_s < \theta$
- A horizontal force M”/r is required to push against the thread to prevent it from sliding downwards
- The magnitude of the moment required to this unwinding is

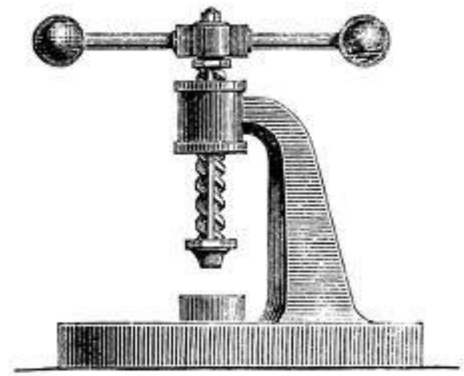
$$\boxed{M'' = rW\tan(\phi_s - \theta)}$$



Downward screw motion ($\theta < \phi_S$)

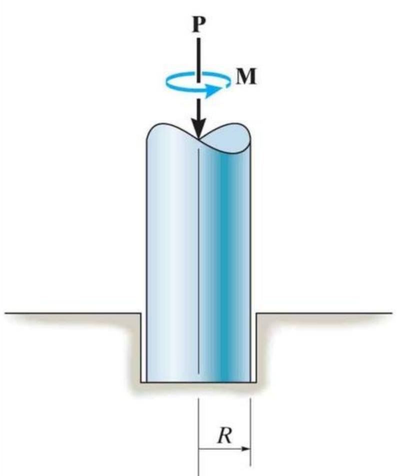
(d)

fig08_16d.jpg



Pivot & Collar Bearings

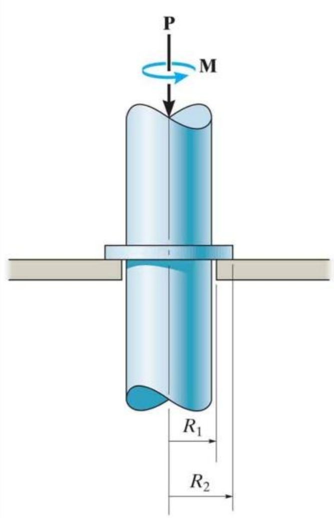
- *Pivot* and *collar bearings* are used in machines to support an axial load on a rotating shaft
- In the absence of lubrication, the laws of dry friction may be applied to determine the moment needed to turn the shaft as it supports the axial load.



Pivot bearing

(a)

fig08_20a.jpg



Collar bearing

(b)

fig08_20b.jpg

Pivot & Collar Bearings: Analysis

- If **P** is the axial force, and the contact area is $\pi(R_2^2 - R_1^2)$, then the normal pressure

$$p = \frac{P}{\pi(R_2^2 - R_1^2)}$$

- Consider an infinitesimally small area of collar subjected to normal force dN and its associated frictional force dF

$$dA = (r d\theta).(dr), \qquad dN = p\, dA$$

$$dF = \mu_s\, dN = \mu_s\, \mathrm{p}\, dA = \frac{\mu_s P}{\pi(R_2^2 - R_1^2)} dA$$

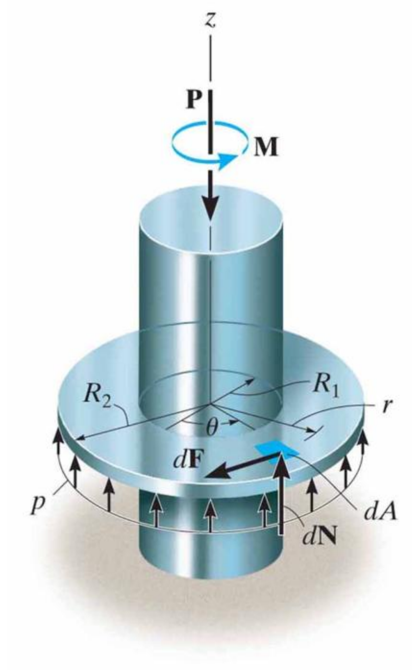


fig08_21.jpg

Pivot & Collar Bearings: Analysis

- The frictional force creates a moment about the z-axis $dM = r\,dF$
- For impending motion

$$\sum M_z = 0; \qquad M - \int_A r dF = 0$$

- Substituting for *dF* and *dA* and integrating over the entire bearing area

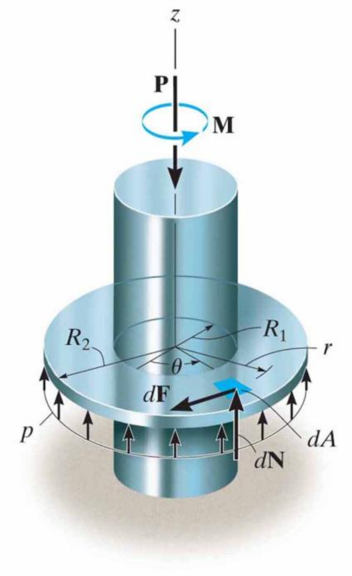


fig08_21.jpg

$$M = \int_{R_1}^{R_2}\int_0^{2\pi} r\left[\frac{\mu_s P}{\pi(R_2^2 - R_1^2)}\right](r\,d\theta\,dr) = \frac{\mu_s P}{\pi(R_2^2 - R_1^2)}\int_{R_1}^{R_2} r^2 dr \int_0^{2\pi} d\theta$$

$$M = \frac{2}{3}\mu_s P\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

Pivot & Collar Bearings: Analysis

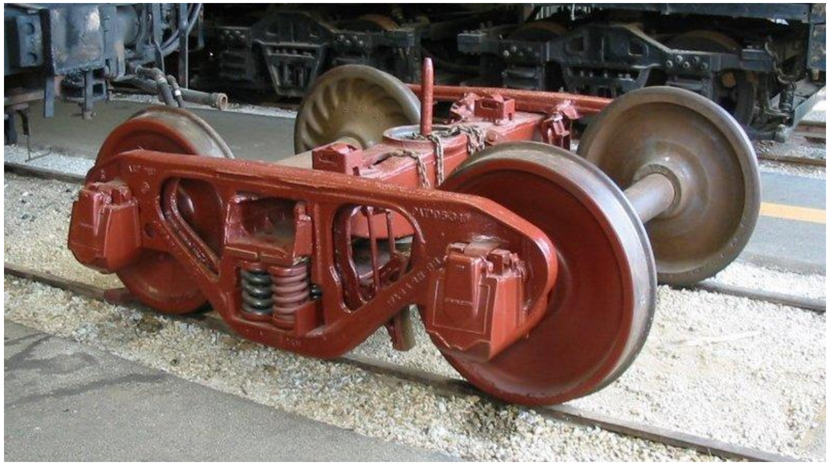
- If the shaft is rotating at constant speed, substitute μ_k for μ_s
- For a pivot bearing $R_2 = R$ and $R_1 = 0$, so

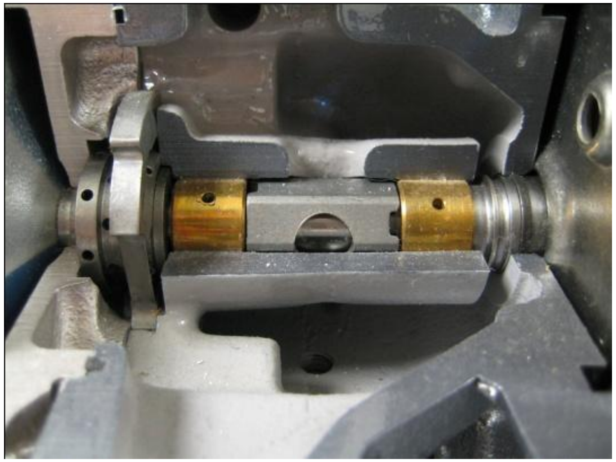
$$M = \frac{2}{3}\mu_s PR$$

- Note that the above equation are based on constant (uniform) pressure on the bearing surfaces.
- If that is not the case a function relating pressure and bearing area must ne determined before integrating

Journal Bearings

- When a shaft or axle is subjected to lateral loads , a journal bearing is used for support.
- In the absence of lubrication we can apply the laws of dry friction



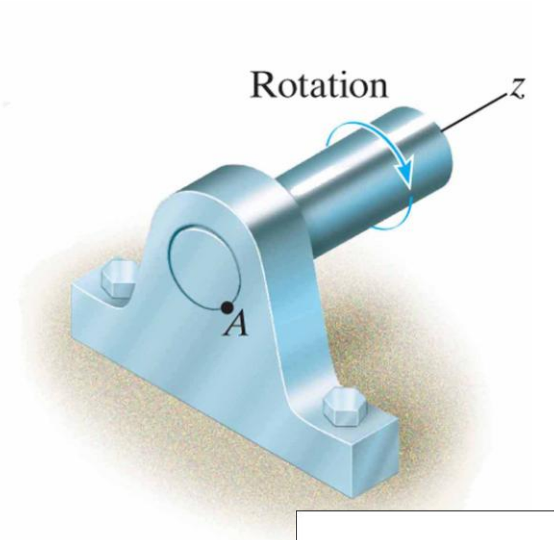


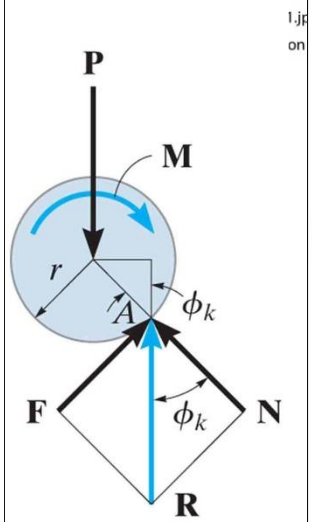
Journal Bearings: Analysis

- Consider the journal bearing support shown
- If the shaft is subjected to a vertical force, resulting in a reactive force at A,
- the moment needed to maintain constant rotation about the z-axis can be found from

$$\sum M_z = 0; \qquad M - (R\sin\phi_k)r = 0$$

$$M = Rr\sin\phi_k$$





(b)

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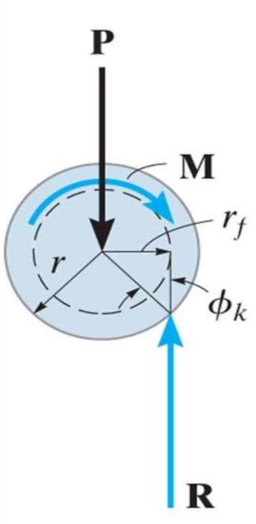


(c)

fig08_23c.jpg

Journal Bearings: Analysis

Where ϕ_s is the angle of kinetic friction, and

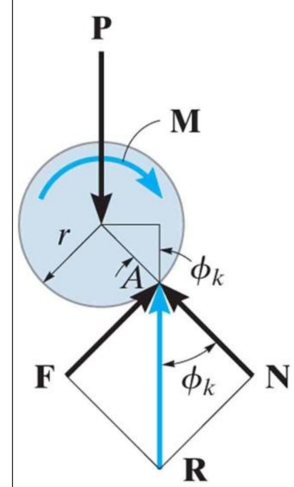
$$\tan\phi_k = F/N = \mu_k N/N = \mu_k$$

- From diagram it can be seen that

$$r\sin\phi_k = r_f$$

- The dashed circle with radius r_f is called the friction circle and R will always be tangent to it
- If there is some lubrication $\sin\phi_k \approx \tan\phi_k \approx \mu_k$
- And the moment needed to overcome the friction becomes

$$M \approx Rr\mu_k$$



(b)

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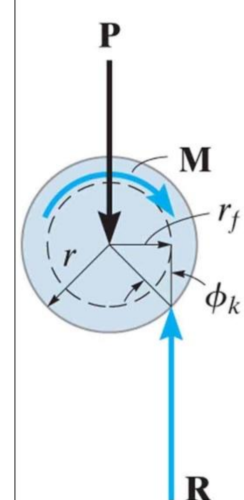


(c)

fig08_23c.jpg

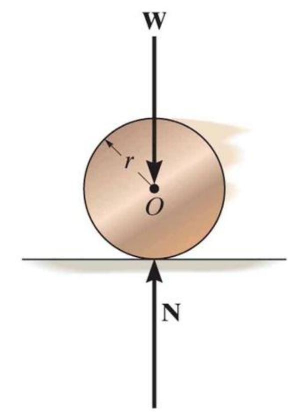
Journal Bearings: Analysis

- In practice journals are prone to rapid wear due to friction between shaft and bearing.
- In engineering design this is overcome by incorporating ball bearings or rollers in the journal bearing to reduce frictional losses



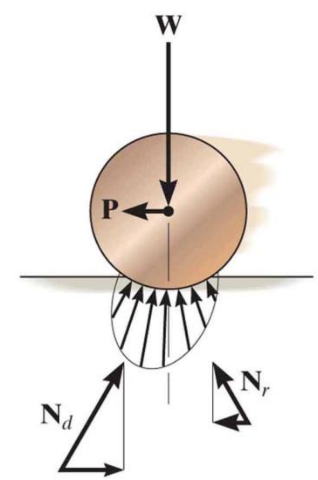
Rolling Resistance

- Consider a *rigid* cylinder rolling at constant velocity over a *rigid* surface.
- Now if the rigid cylinder rolls over a softer surface , it deforms the material of the surface in front of it.
- The reaction of the surface on the cylinder retards its forward motion.
- The material at the rear of the cylinder however rebounds and pushes the cylinder forward



Rigid surface of contact

(a)



Soft surface of contact

(b)

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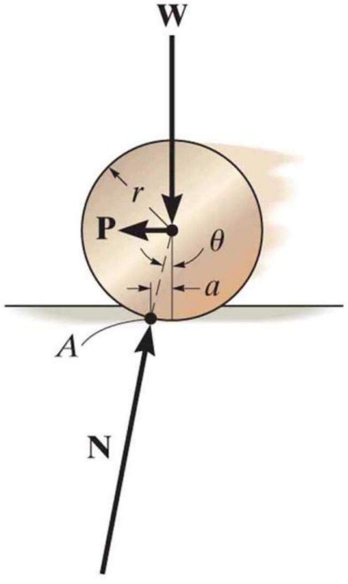
Rolling Resistance

- The deformation force always exceeds the restoration/ rebound force, hence the resistance to rolling.
- The force required to maintain the motion at that constant velocity is

$$P \approx \frac{Wa}{r}$$

where a is the coefficient of rolling resistance



(d)

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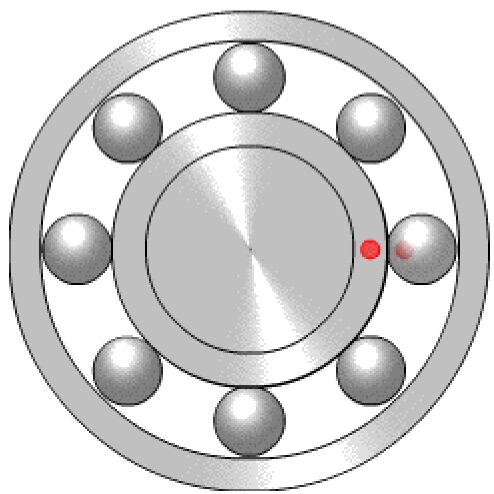
Rolling Resistance

- Rolling resistance is significantly lower than sliding resistance
- Also for a given body, the harder the surface the less the rolling resistance
- The above explain why rollers or ball bearings can be used to minimize friction between moving part of machines



Comments & Questions