Center of Gravity & Centroid

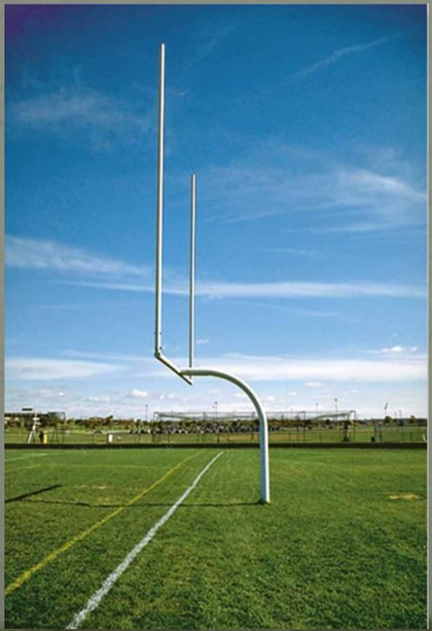
Chapter 9

Overview

- Center of Gravity, Center of mass, Centroid
- Composite Bodies
- Theorems of Pappus and Guldinus
- Resultant of a General Distributed Loading
- Fluid Pressure

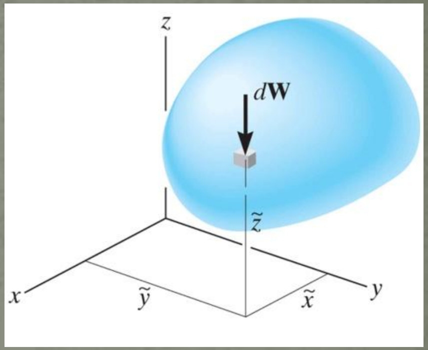
Applications

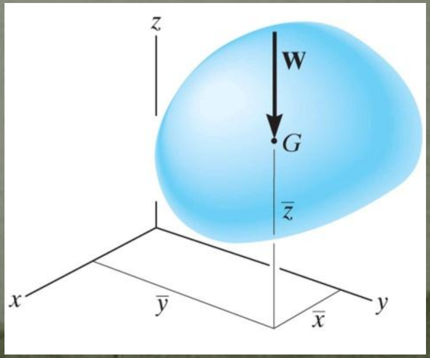
- Location of line of action of forces on structure
- Stability of structures, vehicles etc



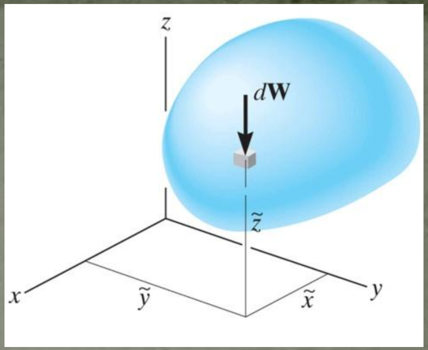
Center of Gravity (CG)

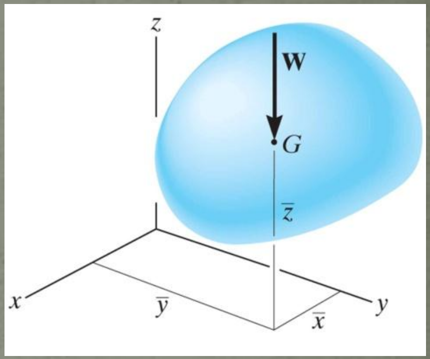
- A body is composed of an infinite number of particles
- The ***center of gravity (CG)*** is a point, often shown as G, which locates the resultant weight of a system of particles or a solid body.
- the sum of moments due to individual particle weights about any point is the same as the moment due to the resultant weight located at G.
- the sum of moments due to the individual particle's weights about point G is equal to zero.





Location of the CG





- If dW is located at point (x, y, z), then

$$x\,W = \int x\,dW$$

- Similarly $y\,W = \int y\,dW$, and

$$z\,W = \int z\,dW$$

- Therefore, the location of the center of gravity G with respect to the x, y,z axes becomes

$$\bar{x} = \frac{\int \tilde{x}\,dW}{\int dW} \qquad \bar{y} = \frac{\int \tilde{y}\,dW}{\int dW} \qquad \bar{z} = \frac{\int \tilde{z}\,dW}{\int dW}$$

Center of Mass & Centroid

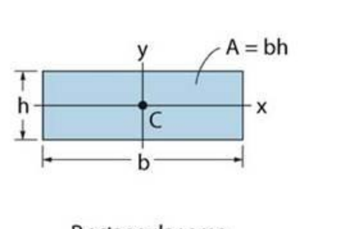
- replacing the *W* with *m* in these equations, the coordinates of the center of mass can be found

$$\bar{x} = \frac{\int \tilde{x}\, dm}{\int dm} \qquad \bar{y} = \frac{\int \tilde{y}\, dm}{\int dm} \qquad \bar{z} = \frac{\int \tilde{z}\, dm}{\int dm}$$

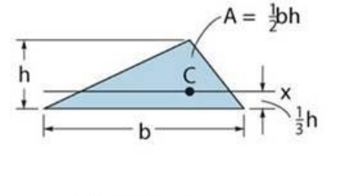
- Similarly, the coordinates of the *centroid* of volume, area, or length can be obtained by replacing *W* by *V*, *A*, or *L*, respectively

Concept of Centroid

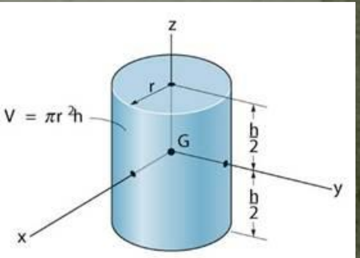
- The *centroid*, C, is a point which defines the geometric center of an object
- The centroid coincides with the center of mass or the center of gravity only if the material of the body is homogenous (density or specific weight is constant throughout the body).
- If an object has an axis of symmetry, then the centroid of object lies on that axis.
- In some cases, the centroid is not located on the object



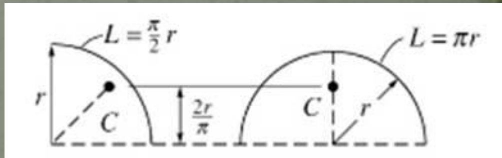
Rectangular area



Triangular area



Cylinder



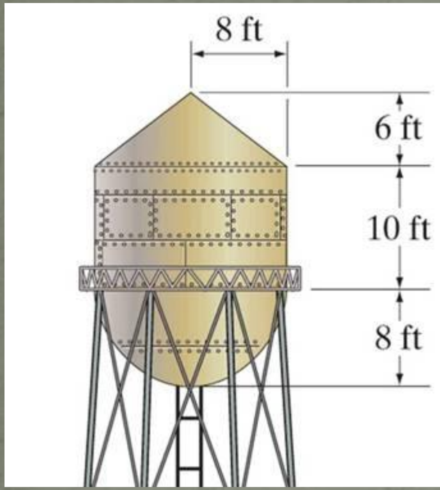
Quarter and semicircle arcs

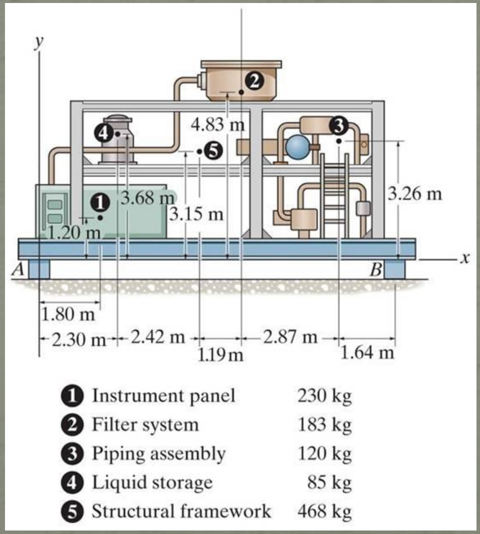
Steps to Determine Centroid of an Area

1. Choose an appropriate differential element dA at a general point (x,y).
2. Express dA in terms of the differentiating element dx (or dy).
3. Determine coordinates (x , y) of the centroid of the rectangular element in terms of the general point (x,y)
4. Express all the variables and integral limits in the formula using either x or y depending on whether the differential element is in terms of dx or dy, respectively, and integrate.
5. Similar steps are used for determining the CG or CM

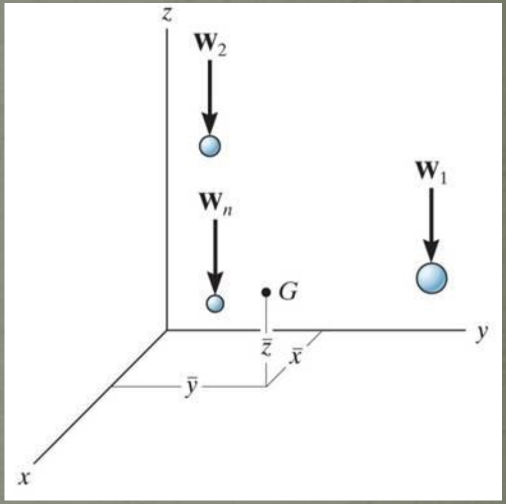
Questions & Comments

Composite Bodies





CG/ CN of Composite Body



- Consider a composite body which consists of a series of particles (or bodies) as shown in the figure. The net or the resultant weight is given as $W_R = \sum W$
- Summing the moments about the y-axis, we get

$$x\, W_R = x_1 W_1 + x_2 W_2 + \ldots\ldots\ldots\ldots + x_n W_n$$

where x_1 represents x coordinate of W_1, etc

CG/ CN of Composite Body

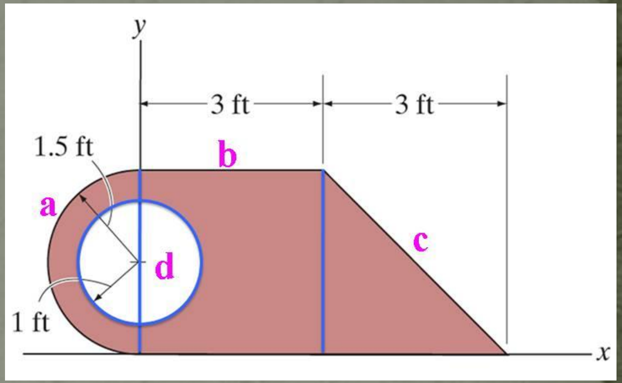
- Similarly, we can sum moments about the x- and z-axes to find the coordinates of G

$$\bar{x} = \frac{\Sigma \tilde{x} W}{\Sigma W} \qquad \bar{y} = \frac{\Sigma \tilde{y} W}{\Sigma W} \qquad \bar{z} = \frac{\Sigma \tilde{z} W}{\Sigma W}$$

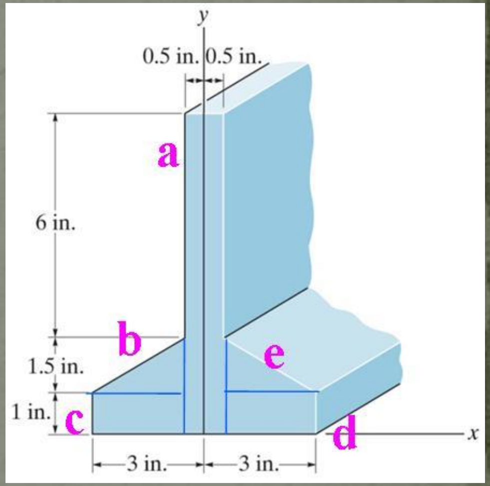
- replacing the W with a M in these equations, the coordinates of the center of mass can be found

What is a Composite Body

- Many industrial objects can be considered as composite bodies made up of a series of connected “simple” *shaped parts* or *holes*



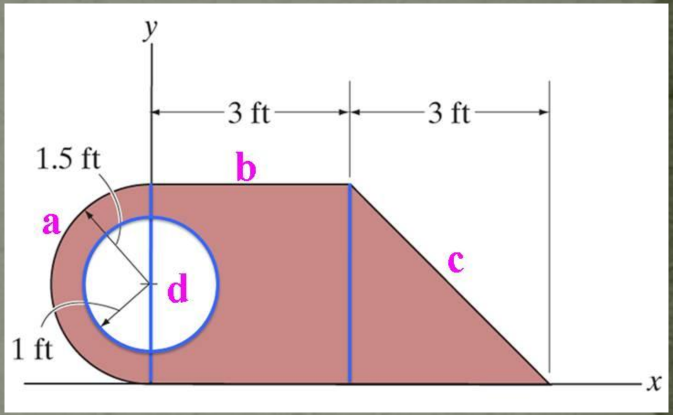
- Knowing the location of the centroid, C, or center of gravity, G, of the simple shaped parts, we can easily determine the location of the C or G for the more complex composite body.

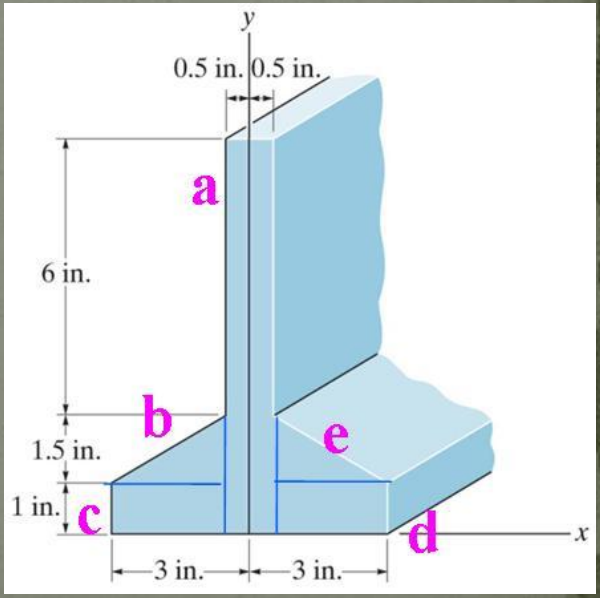


What is a Composite Body

- This can be done by considering each part as a “particle”

- This is a simple, effective, and practical method of determining the location of the centroid or center of gravity of a complex part, structure or machine.



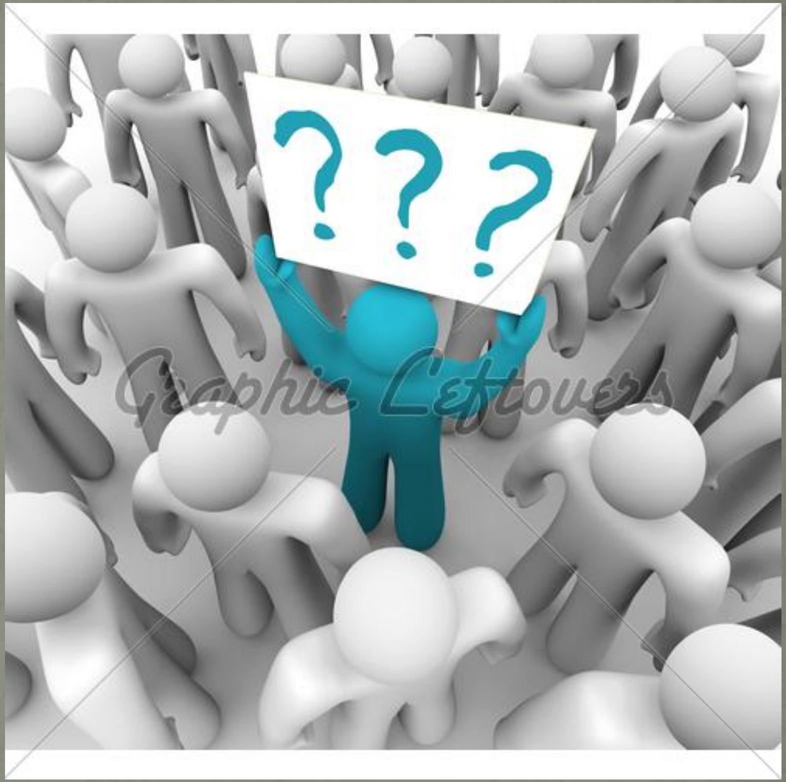


Steps for Analysis

1. Divide the body into pieces that are known shapes. [Holes are considered as pieces with negative weight or size].
2. Make a table compiling weight, mass, or size (depending on the problem), moment arm, and, other columns for recording results of intermediate calculations
3. Fix the coordinate axes, determine the coordinates of the center of gravity of centroid of each piece.
4. Sum the columns to get x, y, and z. Use formulas like

$$x = (\Sigma x_i A_i) / (\Sigma A_i) \text{ or } x = (\Sigma x_i W_i) / (\Sigma W_i)$$

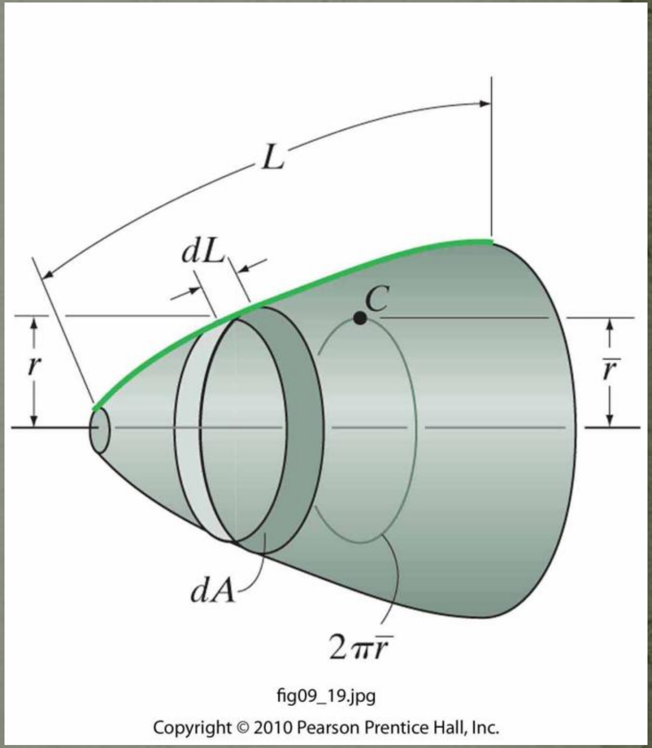
Questions & Comments



Theorems of Pappus & Guldinus

- 1st Theorem: The area of a surface of revolution equals the product of the length of the generating curve and the distance traveled by the centroid of the curve in generating the surface area

$$\boxed{A = \theta \bar{r} L}$$



Where

A = surface area of revolution

θ = angle of revolution in radians, where $\theta \leq 2\pi$

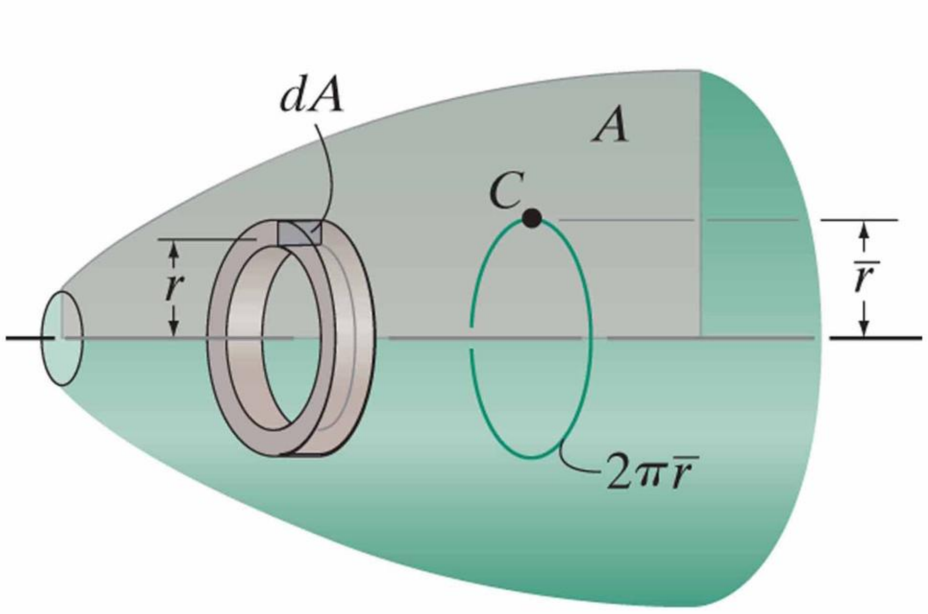
$\bar{r}$ = perpendicular distance from the axis of revolution to the centroid of the generating curve

L = length of generating curve

Theorems of Pappus & Guldinus

- 2nd Theorem: The volume of a body of revolution equals the product of the generating area and the distance traveled by the centroid of the area in generating the volume

$$V = \theta \bar{r} A$$



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Where

V = volume of revolution

θ = angle of revolution in radians,

where $\theta \leq 2\pi$

$\bar{r}$ = perpendicular distance from the axis of revolution to the centroid of the generating area

A = generating area

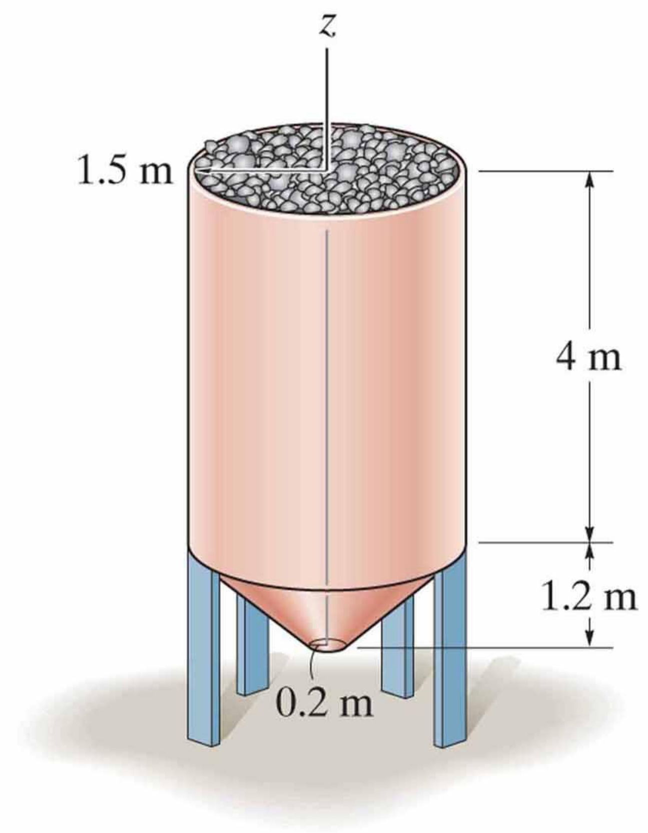
Theorems of Pappus & Guldinus for Composite Shapes

$$A = \theta \sum \left(\tilde{r}\, L \right)$$

And

$$V = \theta \sum \left(\tilde{r}\, A \right)$$

Where $\tilde{r}$ is the perpendicular distance from the axis of revolution to the centroid of each composite part



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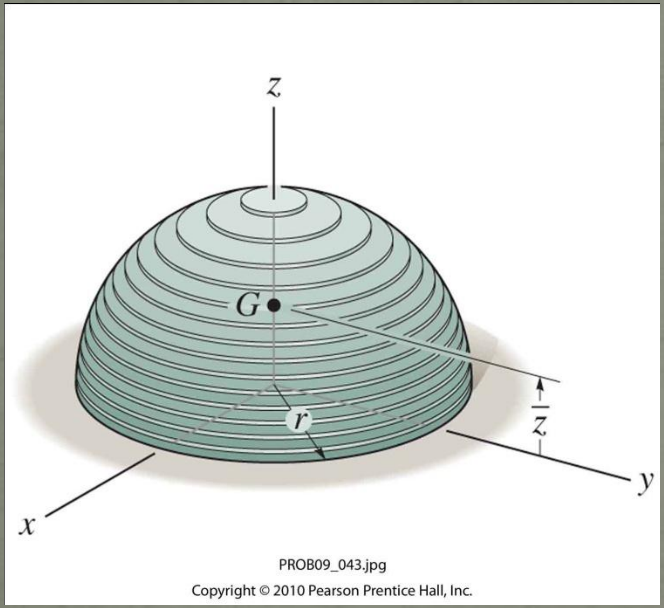
Questions ?


z
G
r
$\bar{z}$
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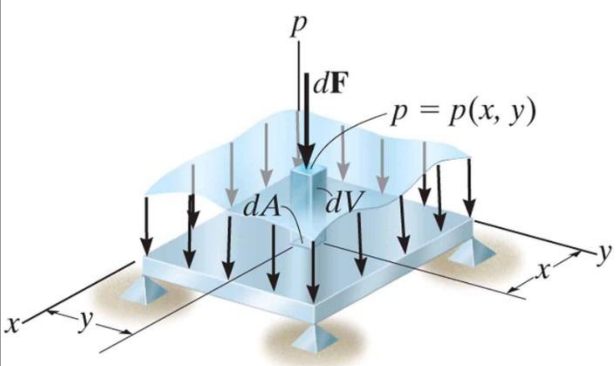
Resultant of a General Distributed Loading

- We shall extend the concept of simplifying a 2-d distributed loading to a single resultant force acting a a specific point, into a 3-d reference frame
- Consider the flat plate subjected to pressure

$$p = p(x, y) = 1\,\text{N/m}^2 \text{ or } 1\,\text{Pa}$$



(a)

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Magnitude of Resultant Force

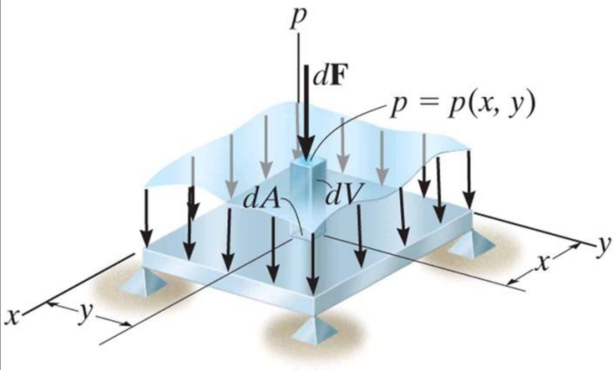
- The force dF acting over the infinitesimally area dA located at some aritrary point (x, y) has magnitude

$$dF = [p(x, y) \,.\, dA\,]$$

- Note that $p(x, y)\,.\,dA = dV$
- The magnitude of the resultant force on the flat plate is the sum of all the differential forces over the entire surface

$$F_R = \sum F;\ \ F_R = \int_A p(x,y)\,dA = \int_V dV = V$$

- So the magnitude of the resultant equals the total volume under the distributed-loading diagram



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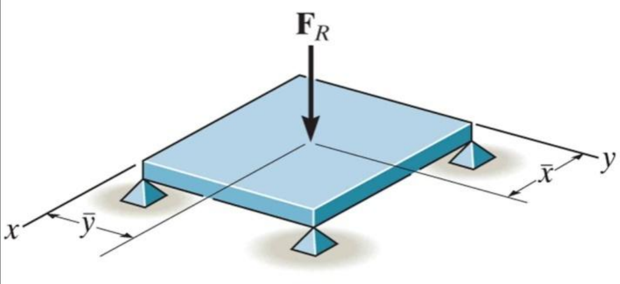

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Location of Resultant Force

- Taking moments of all infinitesimally small forces dF about the x and y axes, the centroid of F_R

$$\bar{x} = \frac{\int_A x p(x, y) dA}{\int_A p(x, y) dA} = \frac{\int_V x dV}{\int_V dV} \qquad \bar{y} = \frac{\int_A y p(x, y) dA}{\int_A p(x, y) dA} = \frac{\int_V y dV}{\int_V dV}$$

- Hence the line of action of the resultant force passes through the geometric center or centroid of the volume under the distributed-loading diagram

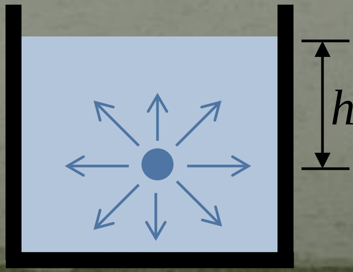
Questions ?

Introduction to Fluid Mechanics

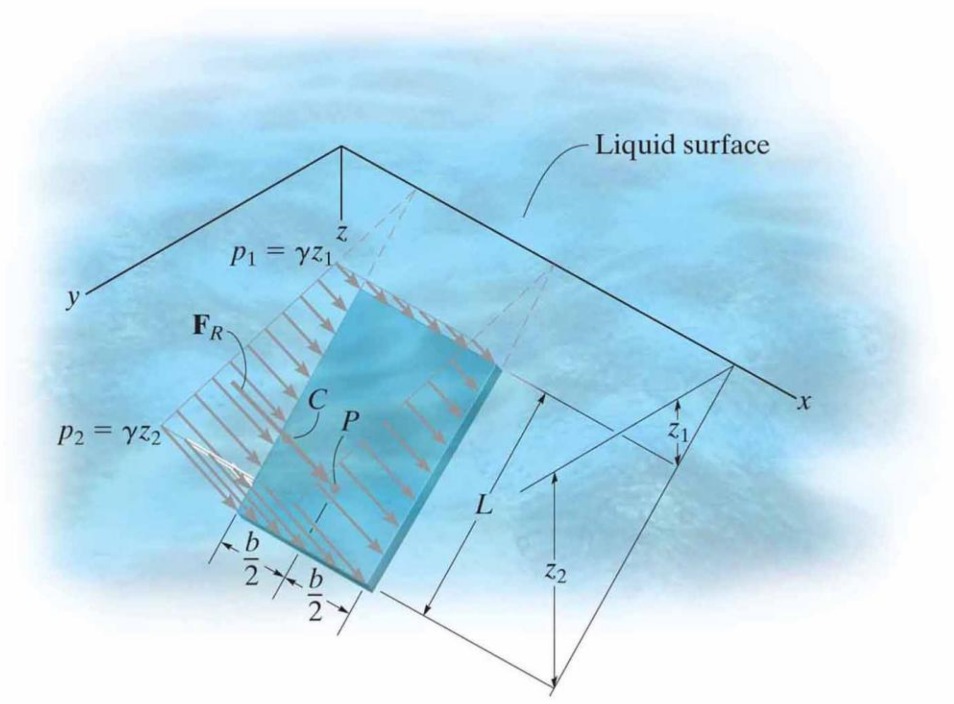
- It was Pascal who observed that the pressure (p) at a point in a fluid is equal in all directions
- For an incompressible fluid, the magnitude of the pressure, as a force per unit area is given by

$$p = h\gamma = h\rho g$$

where
h is the depth of the point from the fluid surface
γ is the specific weight of the fluid
ρ is the mass density of the fluid
g is the acceleration due to gravity



Flat Plate with Constant Width



(a)

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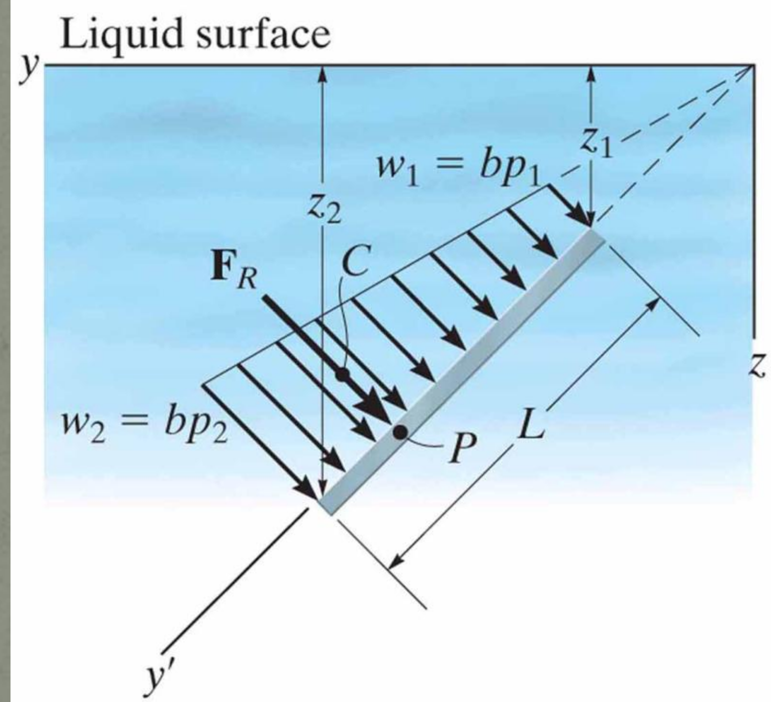

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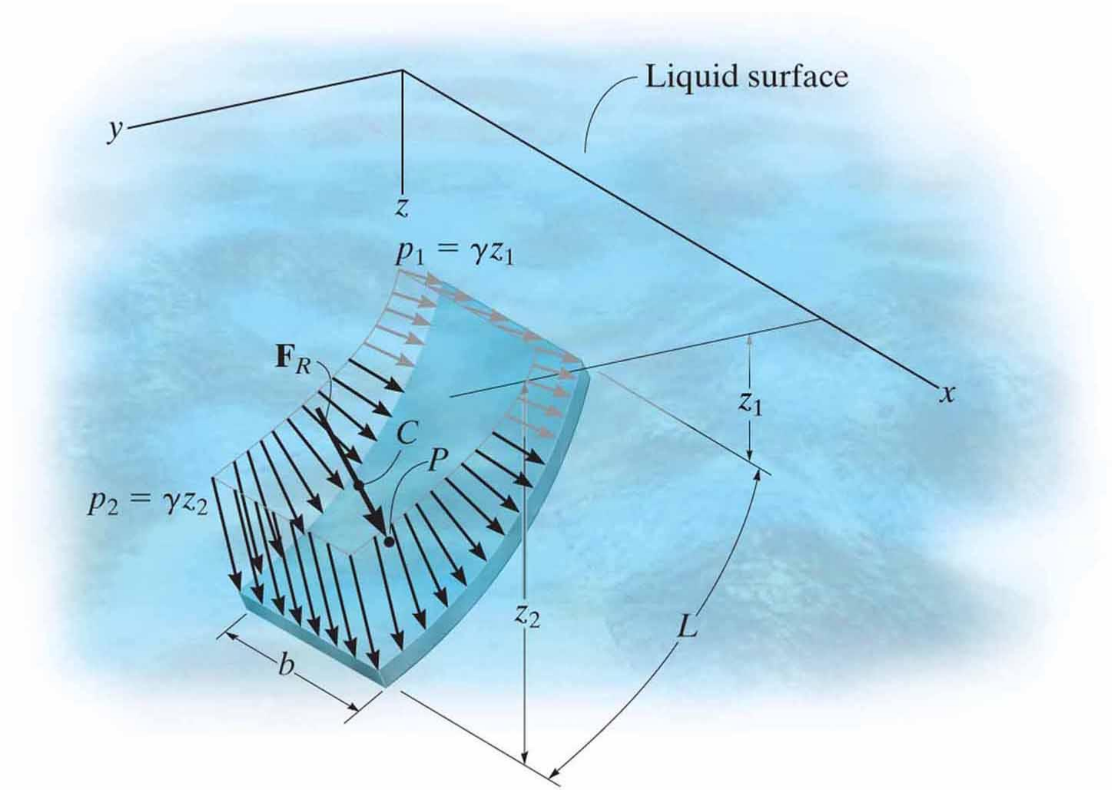
Curved Plate with Constant Width



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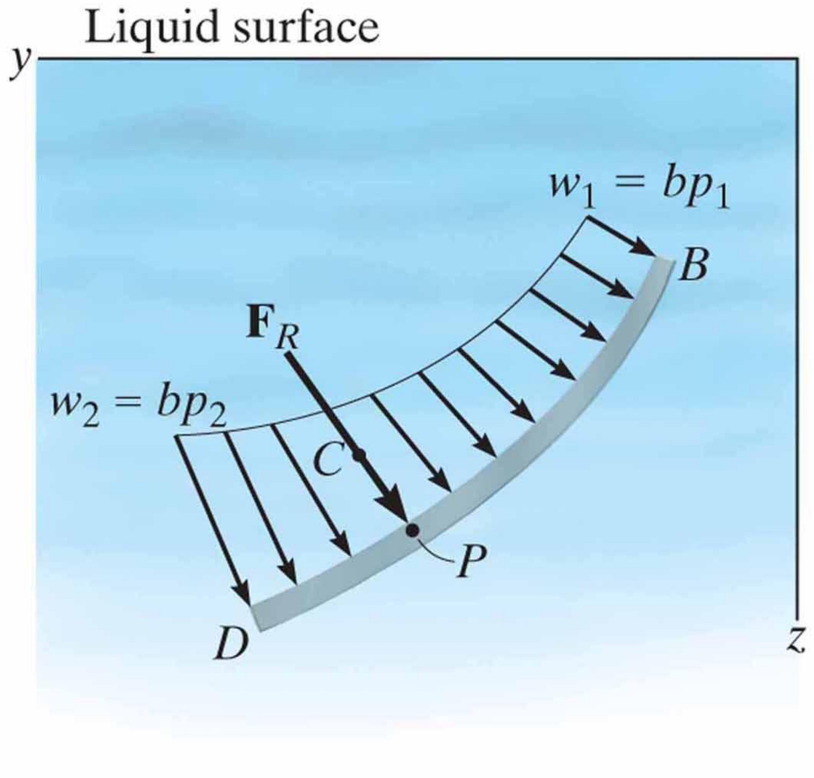
Curved Plate with Constant Width



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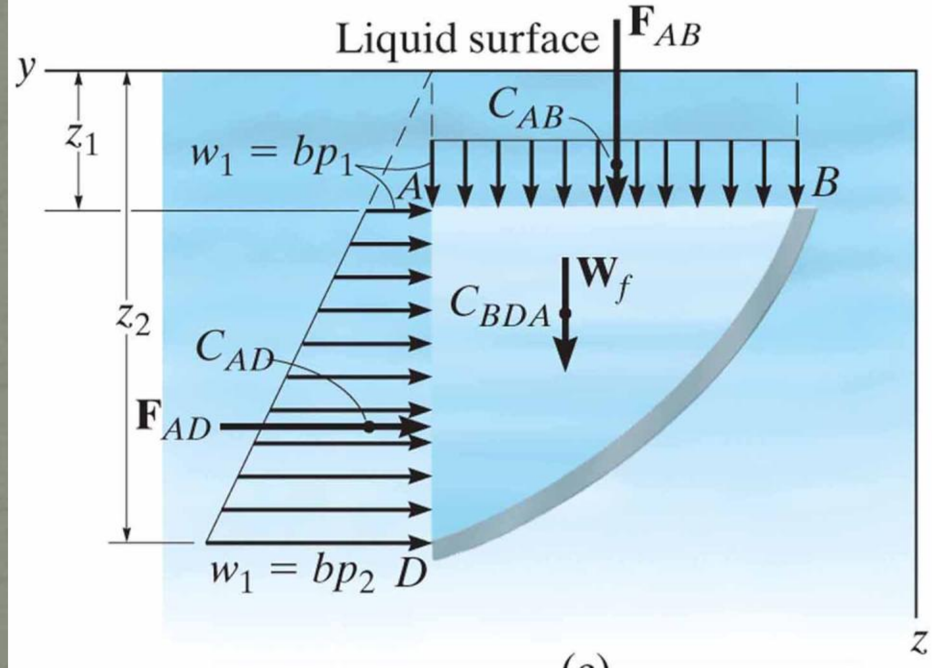

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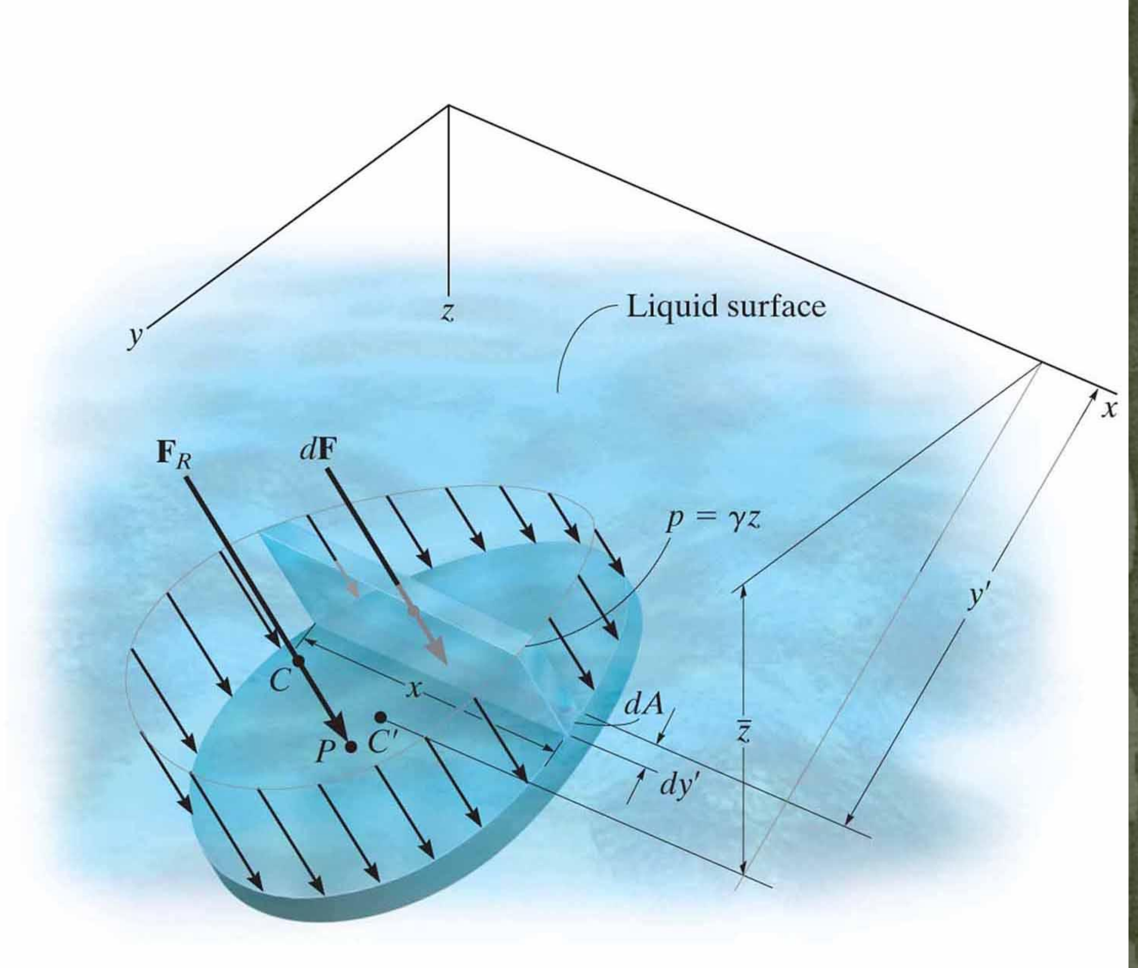
Flat Plate with Variable Width

$$F_R = \int_A dF = \gamma \int_V z\, dA$$

$$F_R = \int_A dF = \gamma \bar{z} A$$



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Questions & Comments