Moment About a Specified Axis

4 – 57

$M_{OA} = u_{OA} \cdot (r \times F)$

$u_{OA} = j$

r = position vector of any point on y-axis (OA) to any point on line of action of F.

$= 200 \cos 45\, i - 50 j - 200 \cos 45\, K$

$= 141.42\, i - 50 j - 141.42\, K$

$F = -16 \cos 30\, i + 16 \sin 30\, K$

$= -13.86\, i + 8\, K$

$$r \times F = \begin{vmatrix} i & j & K \\ 141.42 & -50 & -141.42 \\ -13.86 & 0 & 8 \end{vmatrix}$$

$$= i\left[(-50 \times 8 - 0)\right] - j\left[141.42 \times 8 - (-141.42)(-13.86)\right] + K\left[141.42 \times 0 - (-50)(-13.86)\right]$$

$= -400\, i + 828.72\, j - 693\, K$

$M_{OA} = j \cdot (-400 i + 828.72 j - 693 K)$

$= 828.72$ N mm

Couples

4-74

$\Sigma M_A = 0$

(A is the point of contact of wheel to ground)

$$-500(50) + \underbrace{F(40)}_{\text{couple}} = 0$$

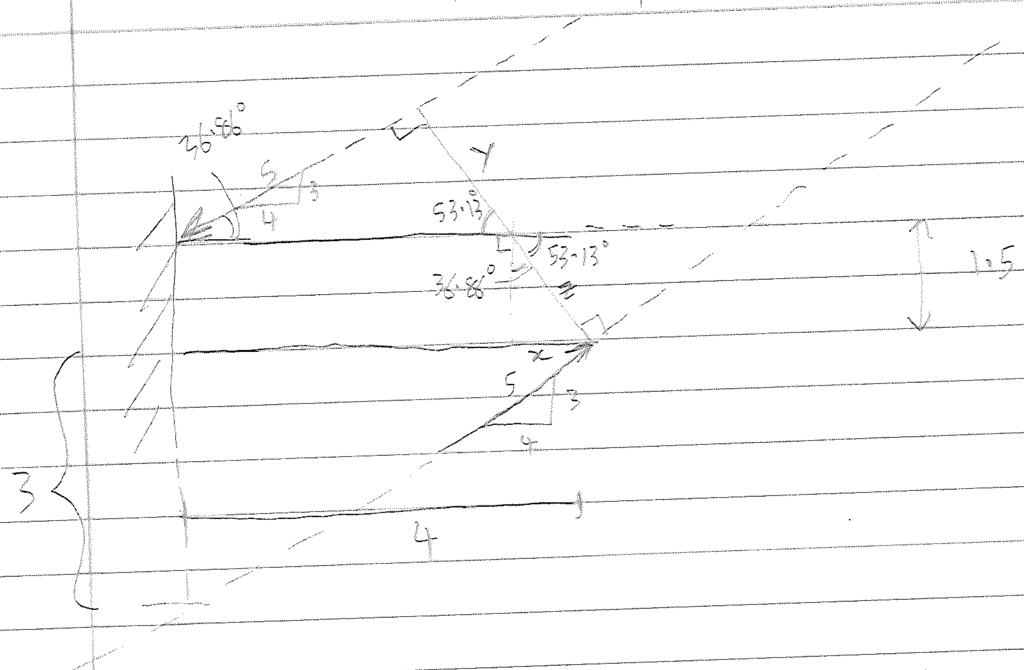
$$F = \frac{500(50)}{40} = 625 \text{ N}$$

4-84

~~Taking moments about left end~~

~~$\Sigma M = \underbrace{-200(1.5)}_{\text{couple}} + \underbrace{Fd}_{\text{couple}} = 300$~~

4 — 84



$$\tan 36.86 = \frac{x}{1.5} \Rightarrow x = 1.5 \tan 36.86 = 1.125$$

$$z^2 = 1.5^2 + (1.5 \tan 36.86)^2$$

$$z = 1.875'$$

$$\sin 36.86 = \frac{Y}{(4 - 1.5 \tan 36.86)}$$

$$Y = \frac{3}{5}\left[4 - 1.5\left(\frac{3}{4}\right)\right] = 1.725'$$

$$\Sigma M = \underset{\text{clockwise}}{-200(1.5)} + \underset{\text{counter clockwise}}{F(1.87 + 1.725)} = 300$$

$$F = 166.89 \text{ lb}$$

Resultant couple acts anywhere on beam b.cos 'free vector'

[Students, confirm using vector formulation!]

Simplification of a Force And Couple System

4 - 111

$$\sum F_x = 500\left(\frac{3}{5}\right) + 200 - 200 = 300\,N$$

$$\sum F_y = -750 + 500\left(\frac{4}{5}\right) = -350\,N$$

resultant force

$$R = \sqrt{300^2 + (-350)^2} = 460.98\,N$$

$$\sum M_0 = -750(1.25) - 500\left(\frac{3}{5}\right).1 + 500\left(\frac{4}{5}\right).(2.5) - 200(1)$$

$$= -437.5\,N \quad \text{(clockwise)}$$

4 - 117

For 3-d problems, vector formulation works better.

$F_1 = 6K$

$F_2 = -5\cos 45 \cdot \sin 30\, i + 5\cos 45 \cos 30\, j + 5\cos 45\, K$

$= -1.77i + 3.06j + 3.53K$

$F_3 = 4\cos 60\, i + 4\cos 60\, j + 4\cos 45\, K$

$= 2i + 2j + 2.83K$

$$\Sigma F = \begin{array}{r} 6K \\ -1.77i + 3.06j + 3.53K \\ 2i + 2j + 2.83K \\ \hline \end{array}$$

$R = 0.23i + 5.06j + 12.36K$

$|R| = \sqrt{0.23^2 + 5.06^2 + 12.36^2} = 13.36$ KN

Moments

$M_O = r \times F$

where r is position from O.

$r_1 = 2i + 6j$

$r_2 = 4i$

$r_3 = 0$

$$M_1 = \begin{vmatrix} i & j & K \\ 2 & 6 & 0 \\ 0 & 0 & 6 \end{vmatrix}$$

$= i[36-0] - j[12-0] + K[0]$

$= 36i - 12j$

$$M_2 = \begin{vmatrix} i & j & K \\ 4 & 0 & 0 \\ -1.77 & 3.06 & 3.53 \end{vmatrix}$$

$$= i[0 - 0] - j[4(3.53) - 0] + K[4(3.06) - 0]$$

$$= -14.12j + 12.24K$$

$$\sum M_0 = 36i - 12j$$

$$\underline{\quad -14.12j + 12.24K}$$

$$36i - 26.12j + 12.24K$$

$$|M_0| = \sqrt{36^2 + (-26.12)^2 + 12.24^2} = 46.13 \text{ KNm}$$

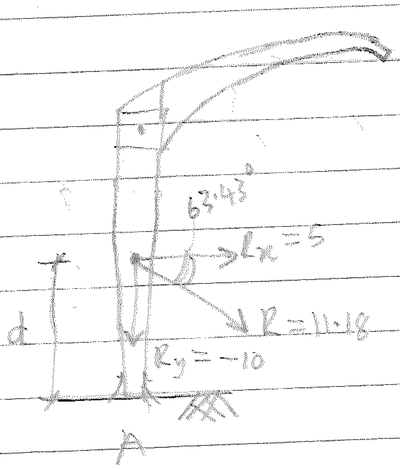
Finding direction cosines will enable us picture its orientation in the 3-d space, by the right hand rule. The question did not ask for this, but, Students, please set it up for your own visualization of your results.

Further Simplification of a Force and Couple System

F4-34

$\sum F_x = 8 - 5\left(\frac{3}{5}\right) = 5$ KN

$\sum F_y = -6 - 5\left(\frac{4}{5}\right) = -10$ KN



So the equivalent moment is caused by the components of R

$R = \sqrt{5^2 + 10^2} = 11.18$ KN

$\theta = \tan^{-1}\left(\frac{-10}{5}\right) = -63.43°$

$\sum M_A = +R_x d = 8(3) - 6(0.5) - 5\left(\frac{4}{5}\right)(2) - 5\left(\frac{3}{5}\right)(4)$

$5d = 1$

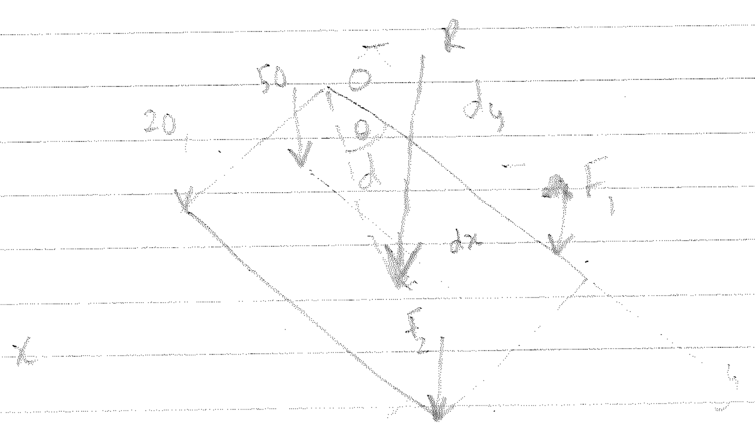
$d = 1/5 = 0.2$ m

4-130

$$\sum F_z = 20 + 50 + 20 + 50 = 140 \text{ kN}$$

(F_1, F_2)

$$\sum M_O = 140d$$



$$\sum M_{y\,axis} = R d_x = 20(10) + 50(4) + 50(10)$$

$$d_x = \frac{900}{140} = 6.43 \text{ m}$$

$$\sum M_{x\,axis} = -R d_y = -50(3) - 20(11) - 50(13)$$

$$d_y = \frac{1020}{140} = 7.28 \text{ m}$$

Note that $d = \sqrt{6.43^2 + 7.28^2} = 9.71 \text{ m}$

We could have used this to take moments about O, and proceeded.