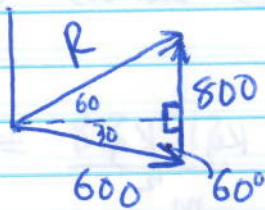
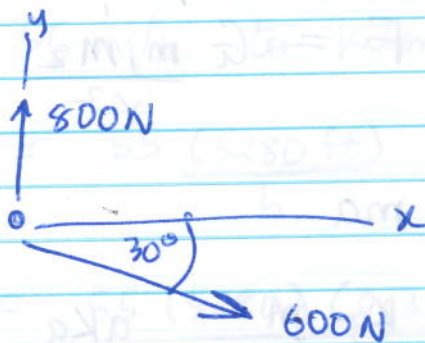


F 2-3

P1-1



I was fixing to set up sine rule, but abandoned it for vertical and horizontal components aka projections

Vertical component of resultant

$$F_v = 800 - 600 \sin 30 = 500 \text{ N}$$

Horizontal component

$$F_H = 600 \cos 30 = 519.62 \text{ N}$$

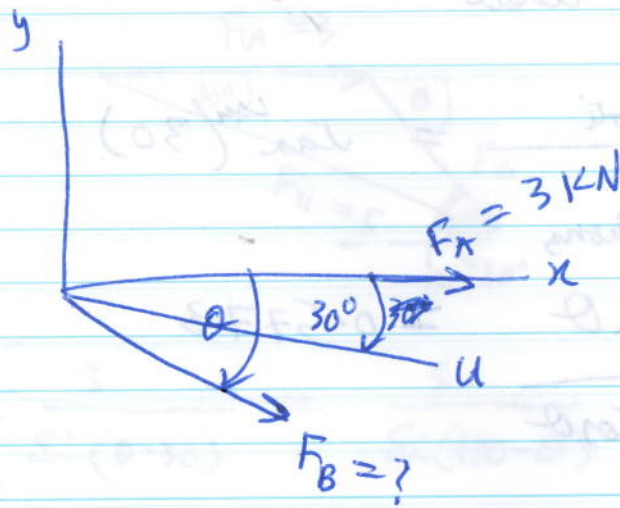
$$|F| = \sqrt{500^2 + 519.62^2} = 721.11 \text{ N}$$

direction: angle $\theta = \tan^{-1} \left(\frac{F_v}{F_H} \right)$

$$= \tan^{-1} \left(\frac{500}{519.62} \right)$$

$$= 43.89^\circ$$

2-8



F_A and F_B act together to produce a resultant

(5) — Let's call resultant vector F_u
 $|F_u| = 5 \text{ kN}$

$$x \text{ component of } F_u = 3 + F_B \cos \theta$$

$$y \text{ component of } F_u = F_B \sin \theta$$

$$|F_u| = \sqrt{(3 + F_B \cos \theta)^2 + (F_B \sin \theta)^2} = 5$$

$$9 + 6F_B \cos \theta + F_B^2 \cos^2 \theta + F_B^2 \sin^2 \theta = 25$$

$$9 + 6F_B \cos \theta + F_B^2 (\underbrace{\cos^2 \theta + \sin^2 \theta}_{=1}) = 25$$

$$9 + 6F_B \cos \theta + F_B^2 = 25$$

$$F_B (6 \cos \theta + F_B) = 16$$

$$6F_B \cos \theta + F_B^2 = 16 \quad \text{--- (1)}$$

we know that

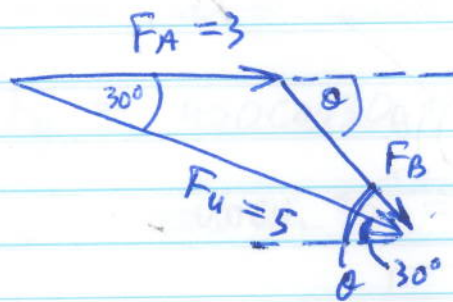
$$\frac{F_{u \text{ verti}}}{F_{u \text{ horiz}}} = \tan(30)$$

$$\frac{F_B \sin \theta}{3 + F_B \cos \theta} = 0.5773$$

$$F_B \sin \theta - 0.5773 F_B \cos \theta = 1.7319 \quad (2)$$

$$F_B (\sin \theta - 0.5773 \cos \theta) = 1.7319$$

2-8



$$\frac{3}{\sin(\theta - 30)} = \frac{5}{\sin(180 - \theta)} = \frac{F_B}{\sin 30}$$

~~$$3 \sin 30 = F_B \sin(\theta - 30)$$~~

$$\frac{5}{\sin(180 - \theta)} = \frac{F_B}{\sin 30}$$

$$\frac{5}{\sin \theta} = \frac{F_B}{\sin 30} \quad \text{--- (1)}$$

Also

$$3 \sin 30 = F_B \sin(\theta - 30)$$

$$= F_B (\sin \theta \cos 30 - \cos \theta \sin 30)$$

$$1.5 = F_B (0.866 \sin \theta - 0.5 \cos \theta) \quad \text{--- (2)}$$

Also

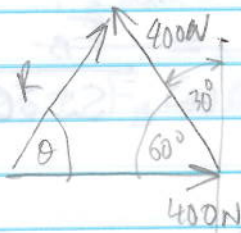
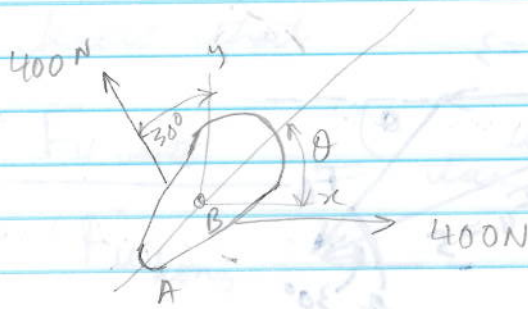
$$3 \sin \theta = 5 \sin(\theta - 30)$$

$$3 \sin \theta = 5 (\sin \theta \cos 30 - \cos \theta \sin 30)$$

$$3 \sin \theta = 5 (0.866 \sin \theta - 0.5 \cos \theta) \quad \text{--- (3)}$$

We have 3 equations, 3 unknowns, solve simultaneously for the unknowns.

2-11



$$\frac{400}{\sin \theta} = \frac{R}{\sin 60} \quad \text{--- (1)}$$

$$\frac{400}{\sin(120-\theta)} = \frac{R}{\sin 60} \quad \text{--- (2)}$$

$$\frac{400}{\sin \theta} = \frac{400}{\sin(120-\theta)} \quad \text{--- (3)}$$

$$\sin \theta = \sin(120-\theta)$$

$$\theta = 120 - \theta$$

$$2\theta = 120$$

$$\theta = 60$$

From (1)

$$\frac{400}{\sin 60} = \frac{R}{\sin 60} \Rightarrow R = 400 \text{ N}$$

Alternately
We can also use projection

$$\sum F_y = 400 \cos 30 = 346.41 \text{ N call it } F_y$$

$$\sum F_x = 400 \sin 30 = 200 \text{ N call it } F_x$$

$$\text{Resultant } |R| = \sqrt{F_x^2 + F_y^2}$$

$$= \sqrt{346.41^2 + 200^2}$$

$$= 399.999 \text{ N}$$

$$\approx 400 \text{ N}$$

direction (relative to the horizontal)

$$\alpha = \tan^{-1} \left(\frac{346.41}{200} \right) = 59.999 \approx 60^\circ$$

Coplanar Forces

2-34

Components of the resultant

horizontal

$$F_x = F_1 \cos \phi + 500 \cos 60 - 450 \left(\frac{3}{5} \right)$$

$$= F_1 \cos \phi - 20$$

$$F_y = F_1 \sin \phi - 500 \sin 60 - 450 \left(\frac{4}{5} \right)$$

$$= F_1 \sin \phi - 793.01$$

Now

$$R^2 = F_x^2 + F_y^2$$

$$600^2 = (F_1 \cos \phi - 20)^2 + (F_1 \sin \phi - 793.01)^2$$

$$= F_1^2 \cos^2 \phi - 2(20)F_1 \cos \phi + 400$$

$$F_1^2 \sin^2 \phi - 2(793.01)F_1 \sin \phi + 5330.46$$

$$= F_1^2 - 2F_1(20 \cos \phi + 793.01 \sin \phi) + 5730.46$$

Components of resultant given

$$x: R \cos \theta = 600 \cos 30 = F_1 \cos \phi - 20$$

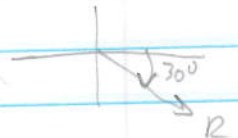
$$\Rightarrow F_1 \cos \phi = 539.62 \quad \text{--- (1)}$$

$$y: -600 \sin 30 = F_1 \sin \phi - 793.01$$

$$\Rightarrow F_1 \sin \phi = 498.01 \quad \text{--- (2)}$$

$$(2) \div (1): \quad \tan \phi = \frac{498.01}{539.62}$$

$$\phi = 42.71^\circ$$

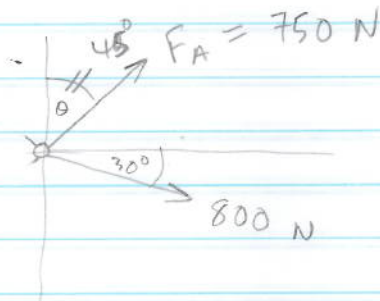


Q2.4

from (1)

$$F_1 = \frac{539.62}{\cos 42.71^\circ} = 734.38 \text{ N}$$

2-48



lets use vector notation

$$x: F_x = 750 \cos 45 + 800 \cos 30 = 1223.15 \text{ N}$$

$$y: F_y = -800 \sin 30 + 750 \sin 45 = 130.33 \text{ N}$$

$$\vec{R} = 1223.15 \hat{i} + 130.33 \hat{j}$$

$$|\vec{R}| = \sqrt{1223.15^2 + 130.33^2} = 1230.07 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{130.33}{1223.15} \right) = 6.08^\circ$$