Force Vector Directed Along A Line

2-91

Use A as our reference point.
Position vector of C (relative to A)

$$r_{AC} = -3i - 6j - 6K$$

$$|r_{AC}| = \sqrt{(-3)^2 + (-6)^2 + (-6)^2} = 9 \text{ m}$$

unit vector in AC direction

$$u_{AC} = \frac{r_{AC}}{|r_{AC}|} = \frac{1}{9}(-3i - 6j - 6K)$$

$$u_{AC} = -\frac{1}{3}i - \frac{2}{3}j - \frac{2}{3}K$$

Force vector for force along AC,

$$F_{AC} = F_C u_{AC} = 600\left(-\frac{1}{3}i - \frac{2}{3}j - \frac{2}{3}K\right)$$

$$F_{AC} = -200i - 400j - 400K$$

For AB:

$$r_{AB} = 4.5\sin 45\, i - 4.5\cos 45\, j - 6K$$

$$= 3.18i - 3.18j - 6K$$

$$|r_{AB}| = \sqrt{3.18^2 + (-3.18)^2 + (-6)^2} = 7.5 \text{ m}$$

$$\cancel{u_{AB} = \frac{1}{7.5}(-200i - 400j - 400}$$

$$u_{AB} = \frac{1}{7.5}(3.18i - 3.18j - 6K)$$

$$u_{AB} = 0.424i - 0.424j - 0.8K$$

$$F_{AB} = 900(0.424i - 0.424j - 0.8K)$$

$$= 381.6i - 381.6j - 720K$$

resultant force acting at A,

$$R_A = F_{AB} + F_{AC} \qquad \text{(all vectors)}$$

$$\begin{aligned} R_A = \;& (-200i - 400j - 400K) \\ + \;& (381.6i - 381.6j - 720K) \\ \hline & 81.6i - 781.6j - 1120K \end{aligned}$$

$$|R_A| = \sqrt{81.6^2 + (-781.6)^2 + (1120)^2}$$

$$= 1368.19 \text{ N or } 1.37 \text{ KN}$$

direction angle

$$\alpha = \cos^{-1}\left(\frac{81.6}{1368.19}\right) = 86.6^\circ$$

$$\beta = \cos^{-1}\left(\frac{-781.6}{1368.19}\right) = 124.8^\circ$$

$$\gamma = \cos^{-1}\left(\frac{1120}{1368.19}\right) = 35.1^\circ$$

Force Directed Along a Line

2-98

Position vector of D relative to B.

$$\vec{r}_{BD} = 2i - 3j - 5.5K$$

unit vector in BD is

$$\vec{u}_{BD} = \frac{\vec{r}_{BD}}{|r_{BD}|} = \frac{1}{\sqrt{2^2 + (-3)^2 + (-5.5)^2}}(2i - 3j - 5.5K$$

$$= 0.3i - 0.46j - 0.84K$$

force vector for F_B

$$\vec{F}_{BD} = F_B \cdot \vec{u}_{BD} = 175(0.3i - 0.46j - 0.84K)$$

$$= 52.5i - 80.5j - 147K$$

Follow procedure for F_A.

Dot Product

2 − 115

F acts along line EB

$r_{EB} = \cancel{4i}\ r_B - r_E$ (position vectors

$= (0i + 2j + 0K) - (4i + 5j - 2K)$

$= -4i - 3j + 2K$

$|r_{EB}| = \sqrt{(-4)^2 + (-3)^2 + 2^2}$

$= 5.39$ m

$u_{EB} = \frac{1}{5.39}(-4i - 3j + 2K)$

$= -0.74i - 0.56j + 0.37K$

$F_{EB} = F_E u_{EB} = 600(-0.74i - 0.56j + 0.37K)$

$= -444i - 336j + 222K$

We want components of F_{EB} along DE and DC. We shall use the Dot Product to do this.

First let us estabish line DE as a vector.

$r_{ED} = r_D - r_E$

$= (4i + 2j - 2K) - (4i + 5j - 2K)$

$= 0i - 3j + 0K$

$u_{ED} = \frac{1}{3} r_{ED} = -j$

Projection of F_{EB} on DE is

$F_{EB} \cdot u_{ED} = (-444i - 336j + 222K) \cdot (-j)$

$= 336$ N

Now, DE and DC are perpendicular

Projection of F_{ED} onto DC (which is $\perp$ to DE)

$$F_{ED/DC} = \sqrt{|F_{ED}|^2 - (F_{EB} \cdot u_{ED})^2}$$

$$\left[A_{\perp} = \sqrt{A^2 - A_{\parallel}^2} \right]$$

$$|F_{ED}|^2 = \sqrt{444^2 + 336^2 + 222^2} = 359316$$

$$F_{ED/DC} = \sqrt{359316 - 336^2} = 496.4 \text{ N}$$

Note that we could have found unit vector for line along DC and repeated to earlier ~~dot~~ product process. A shortcut would be to invoke the property associated with dot product of perpendicular vectors.

Dot Product

2-127

$F_{1x} = +400 \cos 55 . \cos 20 = 215.6\ N$

$F_{1y} = -400 \cos 55 . \sin 20 = -78.5\ N$

$F_{1z} = 400 \cos 35 = 327.7\ N$

[illegible]

$\vec{F_1} = 215.6 i - 78.5 j + 327.7 K$

$|F_1| = \sqrt{215.6^2 + (-78.5)^2 + 327.7^2} = 400$ ok Check

$F_{2x} = 400 \cos 45 = 282.8\ N$

$F_{2y} = 400 \cos 60 = 200\ N$

$F_{2z} = -400 \cos 60 = -200\ N$

$|F_2| = \sqrt{282.8^2 + 2(200)^2} = 399.996$ OK, confirmation

$\vec{F_2} = 282.8 i + 200 j - 200 K$

$\vec{F_1} \cdot \vec{F_2} = |\vec{F_1}| |\vec{F_2}| \cos\theta$

$(215.6)(282.8) + (-78.5)(200) + (327.7)(-200)$

$= |400| |400| \cos\theta$

$\cos\theta =$ [illegible] -0.1267

[illegible] $\theta = 97.27°$