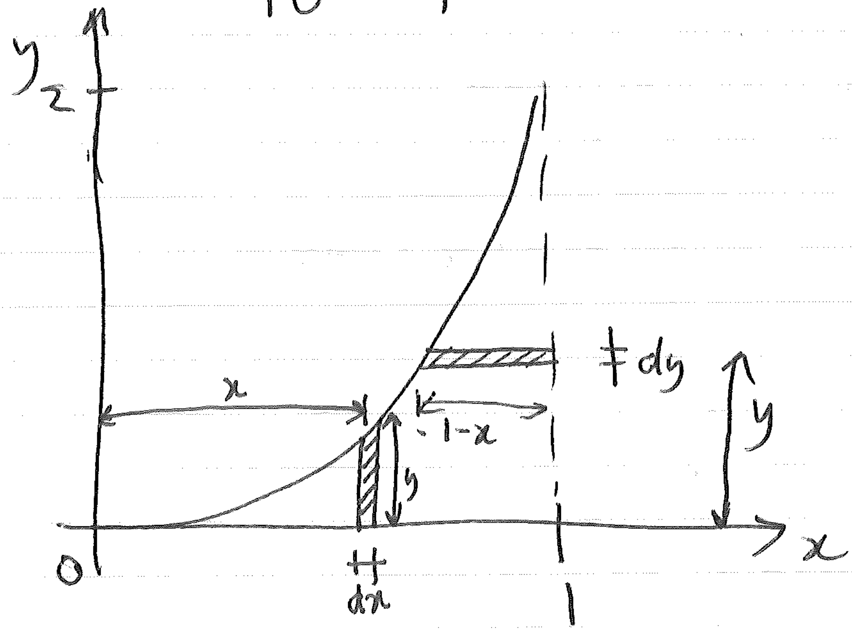
10 − 9



$$dI_x = y^2 dA = y^2(1-x)\,dy$$

$$\int dI_x = \int_0^2 y^2\left[1-\left(\frac{y}{2}\right)^{1/4}\right]dy$$

$$I_x = \int_0^2 \left[y^2 - \left(\frac{1}{2}\right)^{1/4}(y)^{9/4}\right]dy$$

$$= \left[\frac{y^3}{3}\right]_0^2 - \frac{\left(\frac{1}{2}\right)^{1/4}}{13/4}\left[y^{13/4}\right]_0^2$$

$$= \frac{2^3}{3} - \left(\frac{1}{2}\right)^{1/4}\left(\frac{4}{13}\right)\left(2^{13/4}\right)$$

$$= 0.205 \text{ m}^2$$

$$dI_y = x^2 y\,dx$$

$$I_y = \int_0^1 x^2(2x^4)\,dx$$

$$= 2\int_0^1 x^6\,dx$$

$$= \frac{2}{7}\left[x^7\right]_0^1 = \frac{2}{7}(1)$$

$$= 0.286 \ m^2$$

Polar moment of inertia

$$J_O = I_x + I_y$$

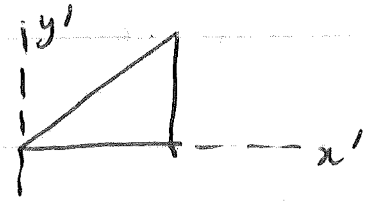
$$= 0.205 + 0.286$$

$$= 0.491 \quad m^4$$

10 – 33

We have 3 basic shapes.

Triangle



$$I_{y'} = \frac{200(300)^3}{36} = 1.5 \times 10^8 \ mm^4$$

By parallel axis theorem

$$I_y = I_{y'} + r^2 A$$

$$r^2 = 200^2 , \ A = \frac{1}{2}(300)(200) = 3 \times 10^4 \ mm^2$$

$$I_y = 1.5 \times 10^8 + 200^2 (3 \times 10^4)$$

$$= 1.35 \times 10^9 \ mm^4$$

Rectangle

$$I_{y'} = \frac{200(300)^3}{12} = 4.5 \times 10^8 \ mm^4$$

$$I_y = 4.5 \times 10^8 + 450^2 (300 \times 200) = 1.26 \times 10^{10} \ mm^4$$

circle

$$I_y = \frac{\pi (75)^4}{4} + (150+300)^2 [\pi (75)^2]$$

$$= 3.6 \times 10^9$$

So moment of inertia of entire composite

$$I = \cancel{(7.2 - 3.6) \times 10^9 \ mm^4}$$

$$ = (1.35 \times \cancel{10^3} + 12.6 - 3.6) \times 10^9 \ mm^4$$

$$\cancel{= 5.4 \times 10^9 \ mm^4}$$

$$= 10.35 \times 10^9 \ mm^4$$