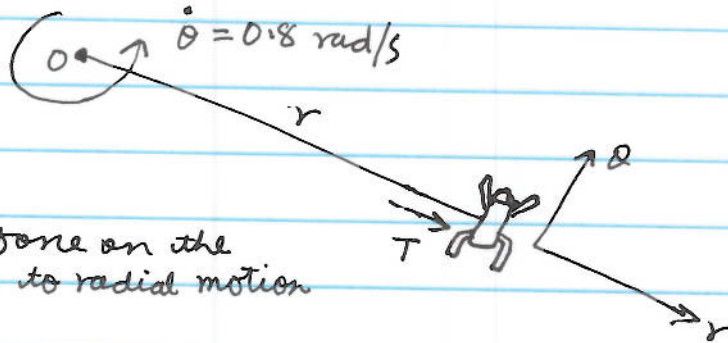


# Equations of Motion: Cylindrical Components

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Looking at system from above



T is the force on the seat due to radial motion

$$\theta = 120^\circ \rightarrow 0.67\pi \text{ radians}$$

this occurs at  $t = ?$

$$\dot{\theta} = 0.8 \Rightarrow \ddot{\theta} = 0$$

by integration

$$\theta = 0.8t$$

$$\text{so } t = \frac{0.67\pi}{0.8} = 2.62 \text{ s.}$$

now working in radians!

$$r = 3 \sin \theta + 5$$

$$= 3 \sin(0.8t) + 5 \quad \text{at } \theta = 2.62 \text{ s } r = 7.59 \text{ m}$$

$$\dot{r} = 3(0.8) \cos(0.8t) \quad \Rightarrow \quad \dot{r} = -1.2 \text{ m/s}$$

$$\ddot{r} = -2.4(0.8) \sin(0.8t) \quad \Rightarrow \quad \ddot{r} = -1.66 \text{ m/s}^2$$

$$\sum F_r = m a_r, \quad a_r = \ddot{r} - r \dot{\theta}^2$$

~~$$T = 20[-0.07 - 5.11(0.67)^2]$$~~

$$T = 20(-1.66 - 7.59 \cdot 0.8^2) = 130.35 \text{ N}$$

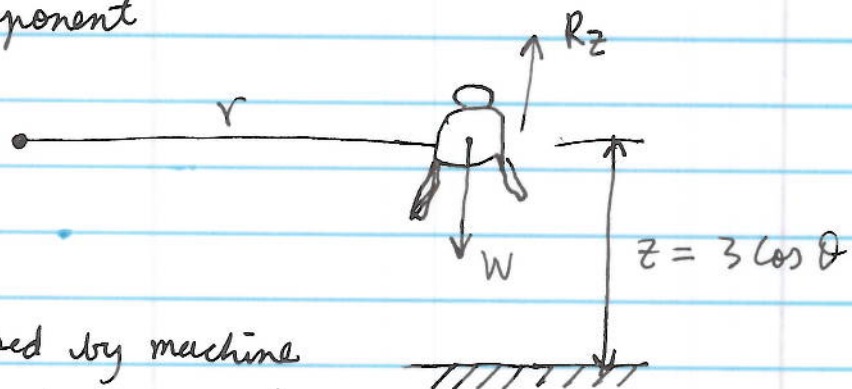
$\theta$ -component

$$\sum F_{\theta} = ma_{\theta}, \quad a_{\theta} = r\ddot{\theta} + z\dot{\theta}$$

let force in  $\theta$ -component exerted on seat be denoted  $F_{th}$

$$\begin{aligned} F_{th} &= m(r\ddot{\theta} + z\dot{\theta}) \\ &= 20 [ 7.59(0) + 2(-1.2)(0.8) ] \\ &= -38.4 \text{ N} \end{aligned}$$

$z$ -component



$R_z$  is caused by machine swinging seat up and down.

$$z = 3 \cos \theta = 3 \cos(0.8t)$$

at  $t = 2.62$

$$z = 3 \cos(0.8 \cdot 2.62) = -1.5 \text{ m}$$

$$\dot{z} = -2.4 \sin(0.8 \cdot 2.62) = -2.07 \text{ m/s}$$

$$\ddot{z} = -1.92 \cos(0.8 \cdot 2.62) = 0.96 \text{ m/s}^2$$

$$\sum F_z = ma_z, \quad a_z = \ddot{z}$$

$$R_z - W = ma_z$$

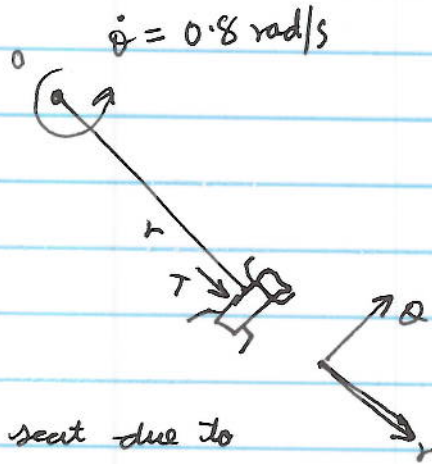
$$R_z - 20(9.81) = 20(0.96)$$

$$R_z = 215.4 \text{ N}$$

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## Alternate Solution Using Degrees

Looking at this system from above



T is force on seat due to radial motion

r-component

$$r = 3\sin\theta + 5 \quad \text{at } \theta = 120 \quad r = 7.59 \text{ m}$$

$$\dot{r} = 3\cos\theta \dot{\theta} \quad \dot{r} = -1.5 \text{ m/s}$$

$$\ddot{r} = -3\sin\theta \cdot \ddot{\theta} + 3\cos\theta \cdot \dot{\theta}^2$$

$$\ddot{r} = 1.5 \text{ m/s}^2$$

$$\ddot{r} = -2.078 \text{ m/s}^2$$

$$\Sigma F_r = m a_r$$

$$T = m a_r, \text{ where } a_r = \ddot{r} - \dot{r} \theta^2$$

$$T = 20 [1.5 - (-1.5)(0.8^2)]$$

$$T = 49.2 \text{ N or } 20(-2.078 - 7.59 \cdot 0.8^2)$$

$$T = -138.712 \text{ N}$$

$\theta$ -component

$$\dot{\theta} = 0.8$$

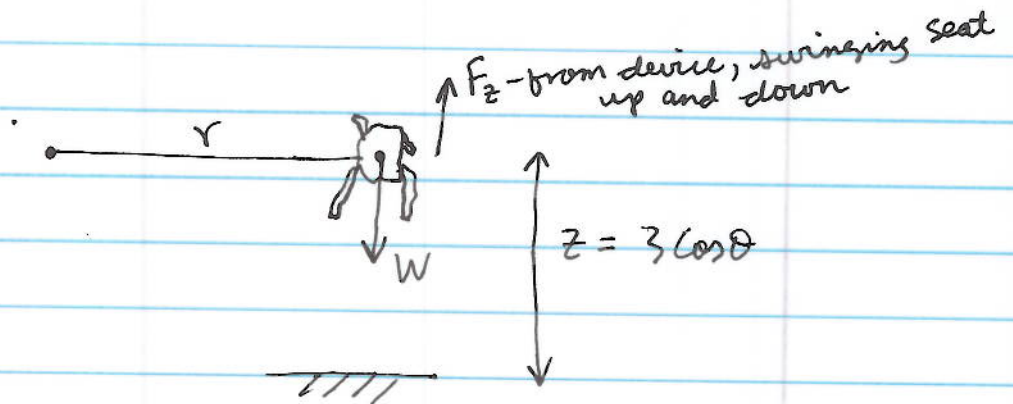
$$\theta = 0.8t$$

$$\ddot{\theta} = 0$$

Students, please check my arithmetic!!



observing system from the side



$$z = 3 \cos \theta$$

$$\dot{z} = -3 \sin \theta \cdot \dot{\theta}$$

$$\ddot{z} = -3 \cos \theta \cdot \ddot{\theta} + (-3 \sin \theta) \cdot \dot{\theta}^2 = -3 \cos \theta \cdot \ddot{\theta}$$

at  $\theta = 120^\circ$

$$z = -1.5 \text{ m}$$

$$\dot{z} = -2.078 \text{ m/s}$$

$$\ddot{z} = 1.2 \text{ m/s}^2$$

$$\sum F_z = m a_z$$

$$F_z - W = m a_z$$

$$F_z - 20(9.81) = 20(1.2)$$

$$F_z = 220.2 \text{ N}$$

$\theta$ -component

$$\sum F_\theta = m a_\theta$$

$$F_\theta = m(r\ddot{\theta} + z\dot{\theta}^2)$$

where  $F_\theta$  is force from machine on seat  
in  $\theta$ -component

$$F_D = 20 [ 7.59.0 + 2.(-1.2)(0.8) ]$$
$$= -38.4 \text{ N}$$

By the way

$\pi \rightarrow \text{USD}$

0.8