P 25 F12-13

for $0 \leq t \leq 5$

$$a = 20$$

$$\frac{dv}{dt} = 20$$

$$\int_0^v dv = 20 \int_0^t dt$$

$$V = 20t$$

for $t \geq 5$ so at $t = 5$, $V = 20(5) = 100 \text{ m/s}$

$$a = -10$$

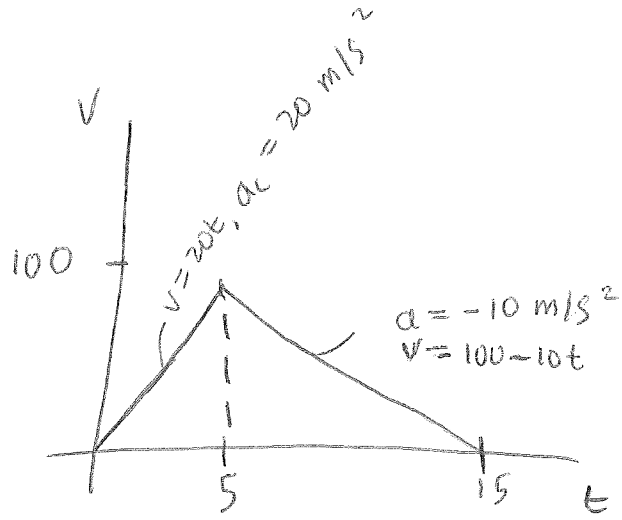
$$\int_{100}^{0} dv = -10 \int_5^{t'} dt$$

$$[0 - 100] = -10[t' - 5]$$

$$-100 = -10(t' - 5)$$

$$10 = t' - 5$$

$$t' = 15 \text{ s}.$$



p. 25 F12-14

$$v = \frac{ds}{dt}$$

$$ds = v\,dt$$

$$\int_0^{s} ds = \int_0^{15} v\,dt$$

$$s = \int_0^{5} 30t\,dt + \int_5^{15} (-15t + 225)\,dt$$

$$= \frac{30t^2}{2}\Big|_0^{5} + \left(-15\frac{t^2}{2} + 225t\right)\Big|_5^{15}$$

$$= \frac{30}{2}(5)^2 + \left(-15\frac{(15)^2}{2} + 225(15) - \left[-15\frac{(5)^2}{2} + 225(5)\right]\right)$$

$$= 375 + \left(-1687.5 + 3375 - \left[-187.5 + 1125\right]\right)$$

$$= 1125 \text{ m}$$

Or simply calculate the area under the piecewise linear function

At $t = 5$, $v = 30(5) = 150$

$$\text{Area} = \frac{1}{2}\cdot 5 \cdot 150 + \frac{1}{2}\cdot 10 \cdot 150 = 1125 \text{ m}$$

P 26 12-43

for $0 \le t < 15$

$$a = \frac{dv}{dt} = 18 \Rightarrow \text{area} \quad v = \int 18\, dt$$

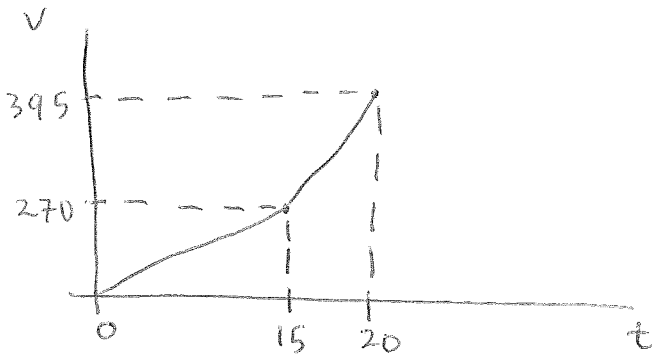
$$\int_0^v dv = \int_0^{15} 18\, dt \Rightarrow v = \left[18t\right]_0^{15} = 270 \text{ m/s}.$$

for $t > 15$

$$\int_{270}^{v} dv = \int_{15}^{20} 25\, dt$$

$$\left[v\right]_{270}^{v} = \left[25t\right]_{15}^{20} \Rightarrow v - 270 = 25(20) - 25(15)$$

$$v = 125 + 270 = 395 \text{ m/s}$$

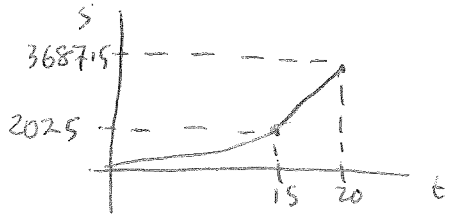


calculate areas under lines to obtain s versus t

$0 \to 15$: Area $= \frac{1}{2} \cdot 15 \cdot 270 = 2025$ m

$0 \to 20$ Area $= 2025 + \frac{1}{2}(270 + 395)(5) = 3687.5$ m

(1662.5)



p. 29 12-59

for $0 \le t \le 5$

$$a = \frac{dv}{dt} = 6t \Rightarrow dv = 6t\,dt$$

$$\int_0^v dv = \int_0^5 6t\,dt \Rightarrow v = 6\frac{t^2}{2}\Big|_0^5$$

$$v = 6\frac{(5)^2}{2} = 75 \text{ m/s}$$

for $t > 5$

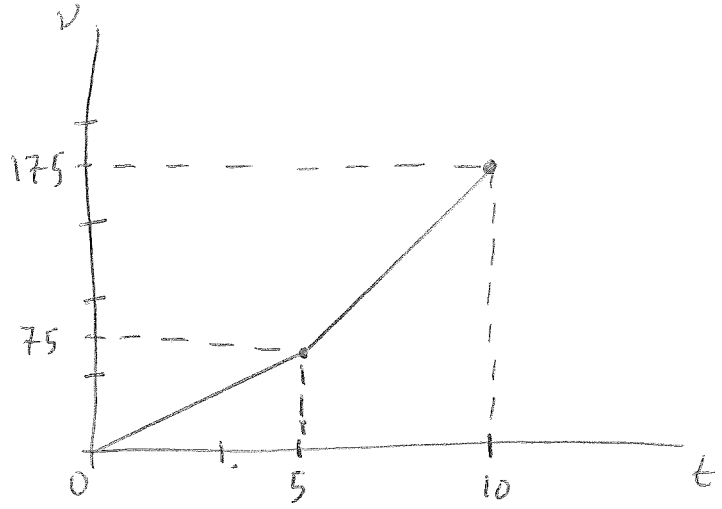
$$a = \frac{dv}{dt} = 2t + 20 \Rightarrow dv = (2t+20)\,dt$$

$$\int_{75}^v dv = \int_5^{10} (2t+20)\,dt \Rightarrow v - 75 = 2\frac{t^2}{2} + 20t\Big|_5^{10}$$

$$v - 75 = \left[t^2 + 20t\right]_5^{10} = 10^2 + 20(10) - \left[5^2 + 20(5)\right]$$

v = 200 m/s

$$v = 100 + 200 - 25 - 100 = 175 \text{ m/s}$$

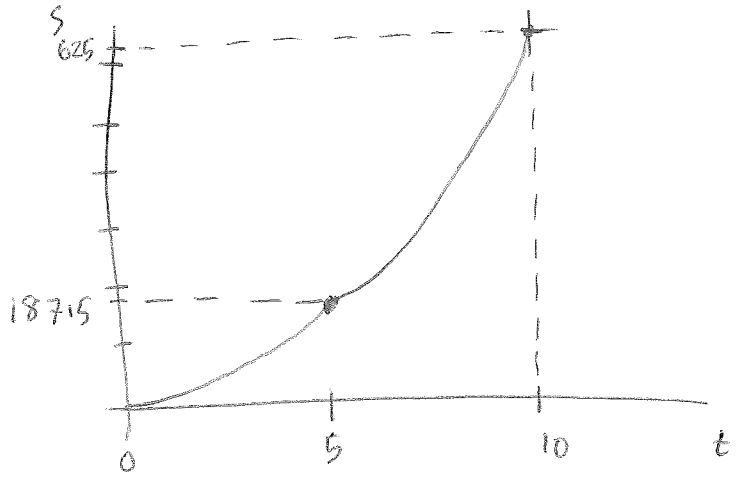


calculate area under curve to obtain distance travelled at these time points in time.

$0 \rightarrow 5$: Area $= \frac{1}{2} \cdot 5 \cdot 75 = 187.5$ m

$0 \rightarrow 10$: Area $= 187.5 + \frac{1}{2}(75 + 175)(5) = 625$ m

(625)



Students: Why are the v-t segments straight line, but the s-t segments curves ??