p. 44   F12-20

$$r = 2\sin(2t)i + 2\cos t j - 2t^2 K$$

$$\frac{dr}{dt} = 2.2.\overset{\cos}{\cancel{\sin}}(2t)i + 2(-\sin t)j - 4tK$$

$$v = 4\cos(2t)i - 2\sin t j - 4tK$$

$$v(t=2) = 4\cos(4)i - 2\sin(2)j - 4(2)K$$

$$= \cancel{4(0.6536)}$$

$$= 4(-0.6536)i - 2(0.9093)j - 8K$$

$$= -2.6144i - 1.8186j - 8K$$

$$a = \frac{dv}{dt} = 4.2.(-\sin 2t)i - 2(\cos t)j - 4K$$

$$= -8\sin(2t)i - 2\cos(t)j - 4K$$

$$\left.\frac{dv}{dt}\right|_{t=2s} = -8\sin(4)i - 2\cos(2)j - 4K$$

$$= -8(-0.7568)i - 2(-0.4161) - 4K$$

$$= 6.0544i + 0.8322j - 4K$$

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$$V = 16t^2 i + 4t^3 j + (5t+2)K$$

$$a = \frac{dV}{dt} = 32t i + 12t^2 j + 5K$$

$$\left.\frac{dv}{dt}\right|_{t=2s} = 32(2)i + 12(2)^2 j + 5K$$

$$= 64i + 48j + 5K$$

$$|a_{t=2}| = \sqrt{64^2 + 48^2 + 5^2} = 80.156 \ m/s^2$$

Position at $t=2$ aka displacement from origin $(0,0,0)$

$$V = \frac{ds}{dt} \Rightarrow ds = V\,dt$$

$$\int_{s=0}^{s=s} ds = \int_{t=0}^{t=2} V\,dt \Rightarrow s = \int_0^2 \left[16t^2 i + 4t^3 j + (5t+2)K\right] dt$$

$$s = 16\frac{t^3}{3} i + 4\frac{t^4}{4} j + \left(5\frac{t^2}{2} + 2t\right)K \Big|_0^2$$

$$= 16\frac{(2)^3}{3} i + (2)^4 j + \left(5\frac{(2)^2}{2} + 2(2)\right)K$$

$$s = 42.667 i + 16 j + 14K$$

p45 F12 – 21, 22

at the high point velocity in the vertical direction is zero

$$V_y = (v_0)_y - gt$$

$$0 = 10 \sin 30 - 9.81\, t$$

$$t = \frac{10 \sin 30}{9.81} = 0.51 \text{ s}$$

this is the time for the high point. We can now use the equation of the trajectory to obtain the magnitude of the high point.

$$y = y_0 + (v_0)_y\, t - \tfrac{1}{2} g t^2$$

$$y_0 = 0$$

$$y = 10 \sin 30\, t - \tfrac{1}{2} \cdot 9.81\, t^2$$

$$\cancel{y = 8.66t - 4.905t^2} \quad y = 5t - 4.905t^2$$

at $t = 0.51$ s

$$y = \overset{5}{\cancel{8.66}}(0.51) - 4.905(0.51)^2 = \overset{1.27}{\cancel{3.44}} \text{ m}$$

Alternately we could have done this in one shot using

$$V_y^2 = (v_0)_y^2 - 2g\,(y - y_0)$$

at high point $V_y = 0$. Also $y_0 = 0$

$$0 = (10 \sin 30)^2 - 2(9.81)\, y$$

$$y = \frac{(10 \sin 30)^2}{2(9.81)} = 1.27 \text{ m}$$

Range is the total horizontal distance till the object hits the ground.

First we need to know the time in flight.

$$y = y_0 + (V_0)_y t - \frac{1}{2} g t^2$$

at impact $y=0$; also $y_0 = 0$

$$0 = 10 \sin 30 t - \frac{1}{2} \cdot 9.81 \cdot t^2$$

$$0 = 5t - 4.905 t^2$$

$$t = \frac{-5 \pm \sqrt{5^2 - 4(-4.905)(0)}}{2(-4.905)}$$ [quadratic formula]

$$t = \frac{-5 \pm 5}{-9.81}$$

$$t = 0, \underline{1.02}$$

So corresponding travel along horizontal

Range: $x = x_0 + (V_0)_x t$

$$x = 0 + (10 \cos 30)(1.02) = 8.83 \text{ m}$$

For velocity at impact we need to V_x and V_y at that point, just before impact, and find the resultant.

$$V_x = (V_0)_x = 10 \cos 30 = 8.66 \text{ m/s}$$

$$V_y = (V_0)_y - gt = 10 \sin 30 - 9.81(1.02) = -5.0 \text{ m/s}$$

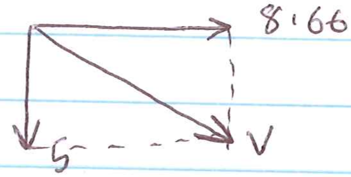
OR

from $V_y^2 = (V_0)_y^2 - 2g(y - y_0)$

$V_y^2 = (10 \sin 30)^2 - 2(9.81)(0-0)$

$V_y = \pm 10 \sin 30 = \pm 5 \text{ m/s}$

Since we know ball is moving downwards at this time, $V_y = -5 \text{ m/s}$



By Pythagoras theorem

$V^2 = 5^2 + 8.66^2 = 99.995 \approx 100$

$V = 10 \text{ m/s}.$

P50 12-98

Let us derive the equation of trajectory. Then derive an equation representing the slope of ground. Using A as the common origin, equate the two equations ~~to~~ and solve.

The equation of the trajectory is given by

$$y = y_0 + (V_0)_y t - \frac{1}{2} g t^2$$

$$= 0 + 40 \sin 30\, t - \frac{1}{2}(9.81)t^2$$

$$y = 20t - 4.905t^2 \quad \text{— (1)}$$

Equation of the ground slope is given by

$$y = \frac{1}{5}x \quad \text{— (2)}$$

Distance covered in the x-plane:

$$x = x_0 + (V_0)_x t$$

$$t = \frac{x - x_0}{(V_0)_x} = \frac{x - x_0}{40 \cos 30} = 0.0288(x - \overset{\sim 0}{x_0})$$

$t = 0.0288x$ This is the same t-value in Eqn (1)

So,

$$y = 20(0.0288x) - 4.905(0.0288x)^2$$

$$y = 0.576x - 0.004068x^2 \quad \text{— (3)}$$

So where (2) and (3) intersect:

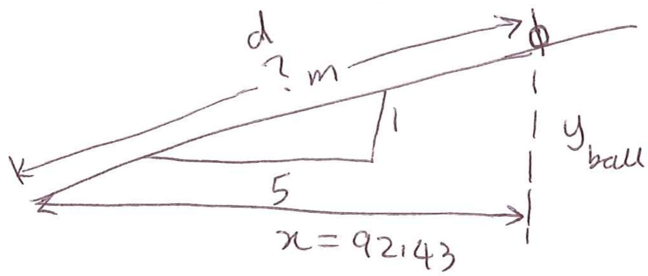
$$\frac{1}{5}x = 0.576x - 0.004068x^2$$

$$0 = 0.376x - 0.004068x^2$$

$$x = \frac{-0.376 \pm \sqrt{0.376^2 - 4(-0.004068)(0)}}{2(-0.004068)}$$

$x = 0$ or 92.43 m

now this x distance is that measured along the horizontal axis. We are being asked for the distance along the slope



using the slope

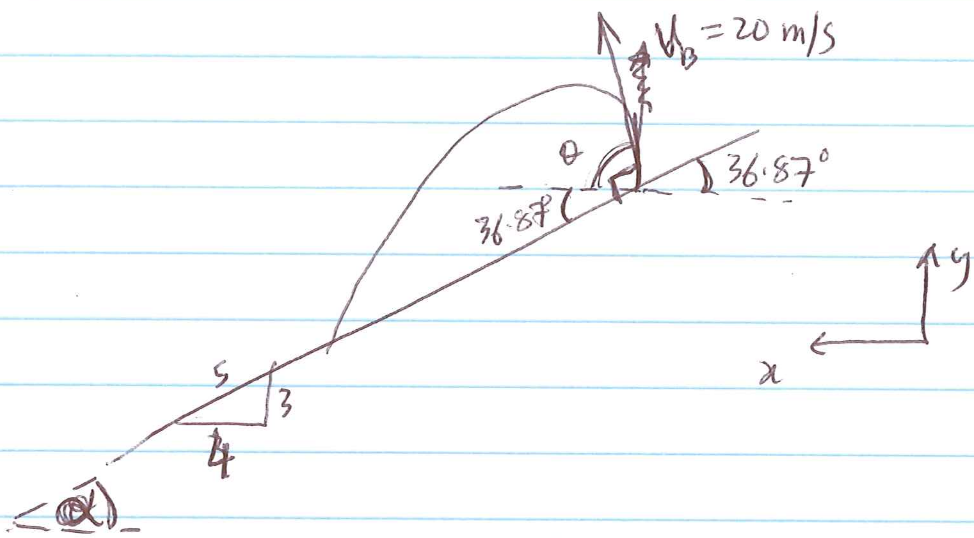
$$y_{ball} = \frac{1}{5} \cdot 92.43 = 18.48 \text{ m}$$

By Pythagoras Theorem

$$d = \sqrt{92.43^2 + 18.48^2} = 94.26 \text{ m}$$

Students, Please check my arithmetic !

F12-24



from 3-4-5 triangle

slope $\frac{3}{4} = \tan\alpha$

$$\alpha = 36.87$$

so launch angle with respect to horizontal

$$= 90 - 36.87 = 53.13°$$

$$y = V_B \sin\theta\, t - \frac{1}{2} g t^2 \quad \text{—(1)}$$

~~$y = t(V_B \sin\theta - \frac{1}{2} g t) \Rightarrow t = V_B \sin\theta - \frac{1}{2} g t$~~

now

$$x = V_B \cos\theta\, t$$

$$\Rightarrow \quad t = \frac{x}{V_B \cos\theta}$$

So for range on slope

$$t = \frac{R\cos\alpha}{V_B \cos\theta} \quad \text{—(2)}$$

so where are we on the trajectory at this time ?.

So if we have a slope of 3 ↑ for every 4 ←,

$$\frac{y}{R} = \sin\alpha$$ (since it is down it is negative)

$$y = -R\sin\alpha \quad \text{——(3)}$$

substituting Eqn 2 and Eqn 3 into Eqn 1.

$$-R\sin\alpha = V_B \sin\theta \cdot \frac{R\cos\alpha}{V_B\cos\theta} - \frac{1}{2}g\left(\frac{R\cos\alpha}{V_B\cos\theta}\right)^2$$

$$-(R\sin\alpha)(V_B\cos\theta)^2 = (V_B\sin\theta)(R\cos\alpha)(V_B\cos\theta) - \frac{1}{2}\cdot 9.81(R\cos\alpha)^2$$

$$-R(\sin 36.87)(20.\cos 53.13)^2 = (20.\sin 53.13)(R\cos 36.87)(20\cos 53.13) - \frac{1}{2}\cdot 9.81(R\cos 36.87)^2$$

$$-86.4R = 153.59R - 3.14R^2$$

$$3.14R^2 - \cancel{67.19}R = 0$$

$$239.99R$$

$$R\left(3.14R - \overset{239.99}{\cancel{67.19}}\right) = 0$$

$$\Rightarrow \quad R = 0, \quad \cancel{3.14R - 67.19 = 0}$$

$$\cancel{R = 21.39 \text{ m}}$$

or

$$3.14R - 239.99 = 0$$

$$R = \frac{239.99}{3.14} = 76.43 \text{ m}$$

QUIZ & Approach to Designing Projectile System.

$$x = (\cancel{v_0})_x t$$

~~$x = $~~

$$t = \frac{x}{V_x} \Rightarrow \boxed{t_R = \frac{R}{V_x} = \frac{R}{V\cos\theta}}$$

at end of flight

$$y = V_y t - \frac{1}{2} g t^2$$

$$0 = t\left(V_y - \frac{1}{2} g t\right)$$

$$\Rightarrow 0 = V_y - \frac{1}{2} g t$$

$$\boxed{t_R = \frac{2V_y}{g} = \frac{2V\sin\theta}{g}}$$

$$\boxed{t_R = \frac{2V\sin\theta}{g} = \frac{R}{V\cos\theta}}$$

$$2\sin\theta\cos\theta = \frac{Rg}{V^2}$$

$$\boxed{\sin 2\theta = \frac{Rg}{V^2}}$$

$$\cancel{\frac{148(32.2)}{V^2}} \not< 1$$

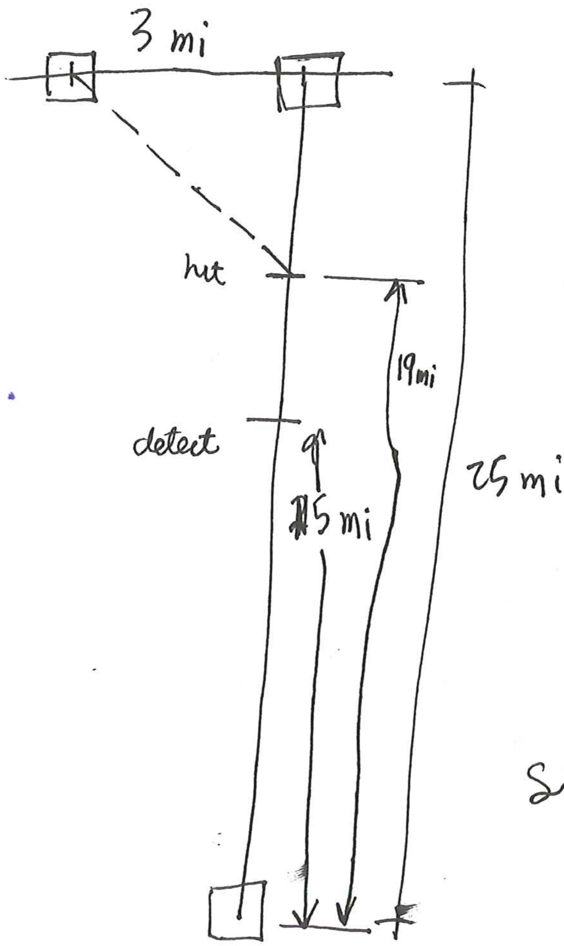
$$\cancel{\frac{V^2}{148(32.2)}}$$

$$0 < \boxed{\frac{Rg}{V^2} < 1}$$

$$\frac{V^2}{Rg} > 1 \quad Rg < V$$

$$V^2 > Rg$$

$$\boxed{V > \sqrt{Rg}}$$



$$\sin 2\theta = \frac{25(5280)(32.2)}{\cancel{[60(1.47)]^2}\ [2100]^2}$$

$$\theta = 37.26^\circ$$

launched at ground level

$u = 2100$ ft/s

time to hit target

$$t = \frac{R}{V\cos\theta} = \frac{25(5280)}{2100\cos 37.26} = 78.97\ \text{s.}$$

anti missile system can detect it 10 miles from target, and respond 10 seconds after detection

$$x = (V_0)_x t$$

$$10(5280) = 2100[\cos(37.26)]t$$

$$t = 31.5 \text{ s.}$$

Time anti system responds $= 31.5 + 10 = 41.5$

At this time missile will be at displacement

$$2100\cos(37.26) \cdot 41.5 = 69362.367 \text{ or } 13.13 \text{ mi}$$

The missile shall be hit at 6 miles from target at that point

$$19(5280) = 2100\cos(37.26) \cdot t$$

$$t = 60.02 \text{ s.}$$

Ht

$$\text{ht}, \; y = (V_0)_y t - \tfrac{1}{2}gt^2$$

$$= 2100\sin(37.26)[60.02] - \frac{32.2}{2}[60.02]^2$$

$$= 18,311.32 \text{ feet.}$$

Lets say anti system on a hill ~~1~~ miles above target (and launch site) (so in beginning we are assuming launch site and target at same level.

It must travel $\sqrt{3^2 + 6^2} = 6.708$ miles
or 35,419.32 ft

Time to get there?

$$35,419.32 = 2000 \cos\theta . t$$

$$t = \frac{35419.32}{2000 \cos\theta}$$

In the y:

$$y = y_0 + (V_0)_y t - \frac{1}{2} g t^2$$

$$35419.32 = 5280 + 2000 \sin\theta . t - 16.1 t^2$$

$$0 = -30139.32 + 2000 \sin\theta . t - 16.1 t^2$$

$$t = -2000 \sin\theta \pm \sqrt{(-2000 \sin\theta)^2 - 4(-16.1)(}$$

Lets say anti-missile has range of 50 mi

$$V \geq \sqrt{50(5280)(32.2)} = 2915.61 \text{ ft/s} \rightarrow 2950 \text{ ft/s}.$$

$$\sin 2\theta = \frac{50(5280)(32.2)}{2950^2} = 0.9768$$

$$2\theta = 77.64$$

$$\theta = 38.82^\circ$$

~~If asking students to figure out launch angle.~~

Time get to intercept point.

$$x = (V_0)_x t$$

$$35{,}419.32 = V\cos\theta \,.\, t \Rightarrow t = \frac{35419.32}{2950\cos\theta} = 15.3\,s$$

$$\cancel{18{,}311.32 = 5280 + 2950\sin\theta\,.\,t - 16.1t^2}$$

$$= \cancel{5280 + 2950\sin\theta \,.\, \frac{35419.32}{2950\cos\theta}} - 16.1$$

$$\cos\theta = \frac{\sqrt{(3\times 5280)^2 + (6\times 5280)^2}}{2950(11.06)} =$$

$$\sin 2\theta = \frac{148(32.2)}{\left[(1.47)(50)\right]^2} = 0.8821$$

$$2\theta = 61.90$$

$$\theta = 30.95^\circ$$

time get these

$$t_R = \frac{148}{(50)(1.47)\cos(30.95)} = 2.34\ s.$$

lets say play action

11.7 s