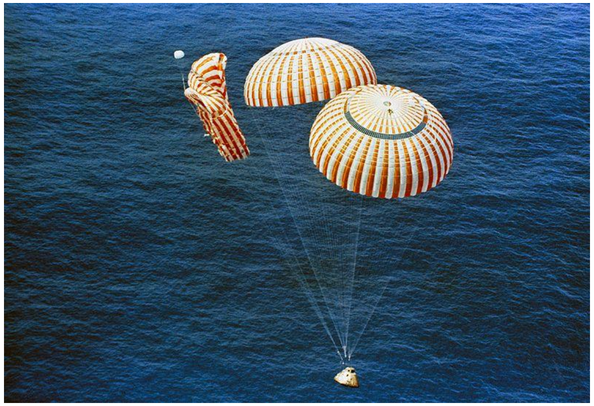
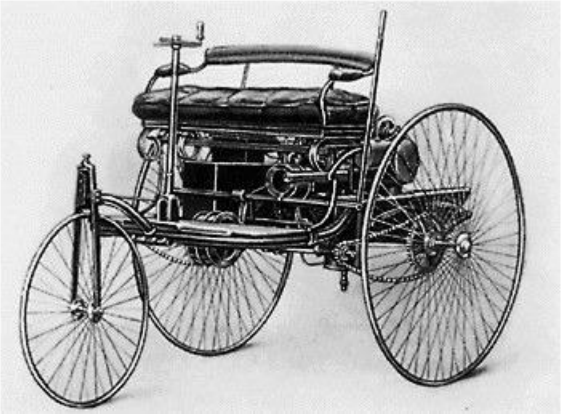
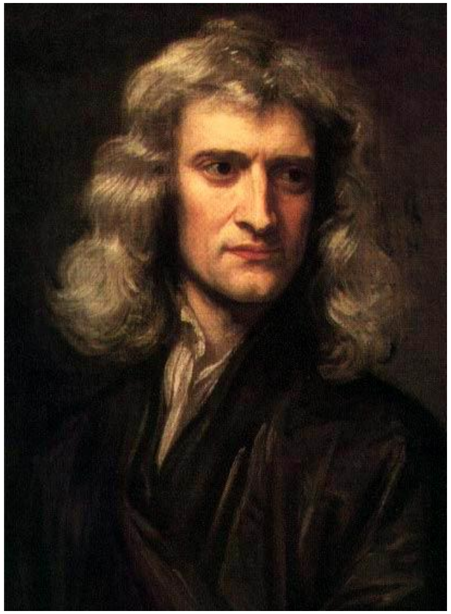




Rectilinear Kinematics

Continuous Motion



Sir Isaac Newton

Leonard Euler

Overview

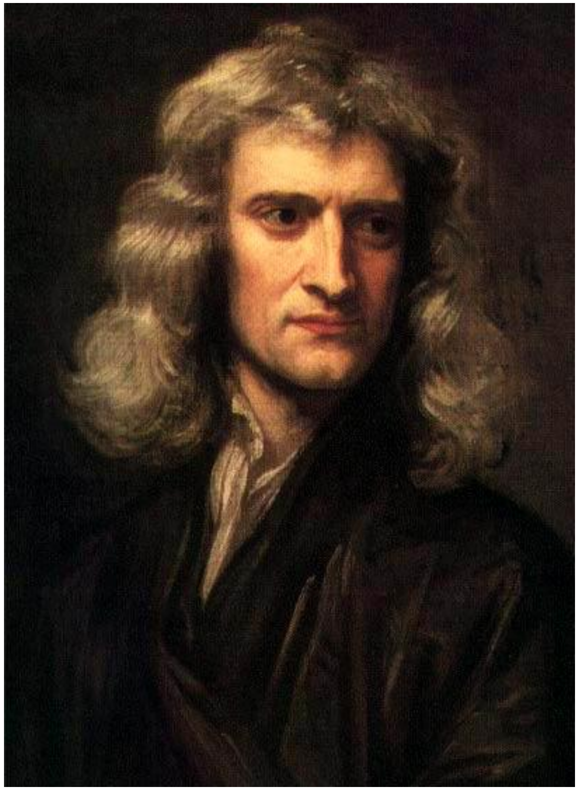
- Kinematics
- Continuous Motion
- Erratic Motion

Michael Schumacher. 7-time Formula 1 World Champion

Kinematics

The objective of kinematics is to characterize the following properties of an object at an instant during its in motion:

- Position
- Velocity
- acceleration



Sir Isaac Newton

Kinematics

- Assumptions in kinematics :
  - the object is negligible size and shape (particle)
  - The mass is not considered in the calculations
  - Rotation of the object is neglected
- We shall look at the kinematics of an object moving in a straight line. We call this **Rectilinear Kinematics**

Rectilinear Kinematics: Continuous Motion

- Consider a particle in rectilinear motion from a fixed origin *O* in the S direction.

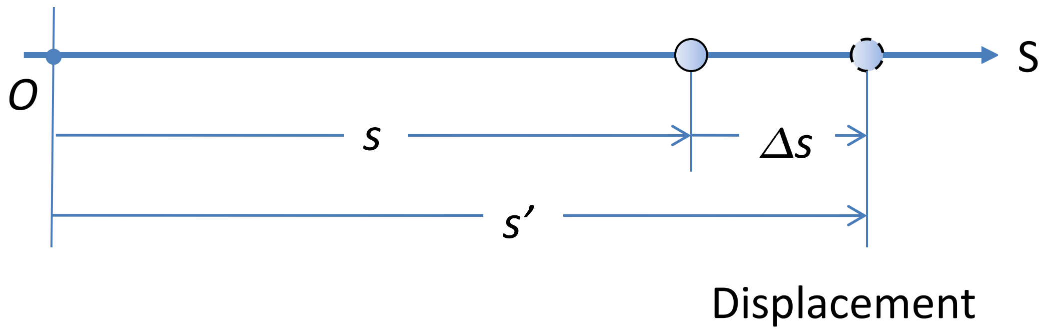
- For a given instant, *s* is the *position coordinate* of the particle
- The magnitude of *s* is the *distance* from the origin (in feet, meters or the relevant unit of measure)

Rectilinear Kinematics: Continuous Motion

- Note that the *position coordinate* would be negative if the particle traveled in the opposite direction according to our frame of reference
- *Position* is has a magnitude (distance from origin) and is based on a specific direction. It is therefore a *vector quantity*

Continuous Motion - Displacement

- *Displacement* is defined as the change in position



$$\Delta s = s' - s$$

Continuous Motion - Displacement

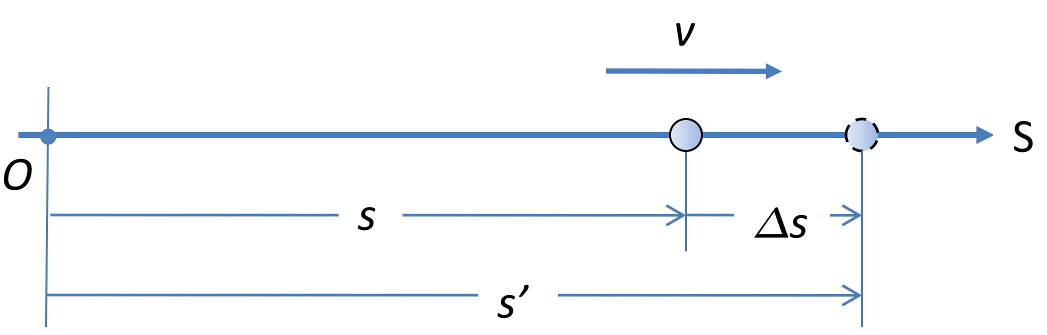
- *Displacement* is also a vector quantity characterized by a magnitude and a direction
- Note that *distance* on the other hand is a scalar quantity representing the length from an origin.

France vs Wales. Six Nations Cup. Lineout

Continuous Motion – Average Velocity

- If the particle undergoes displacement Δs over a time interval Δt, then the *average velocity* over this time interval

$$v_{avg} = \frac{\Delta s}{\Delta t}$$



Velocity

Continuous Motion – Instant. Velocity

- If we were to take smaller and smaller values of Δt, then Δs would also get smaller and smaller.
- At some point would no longer be an interval but a point in the time dimension (instant). The associated velocity is called the *instantaneous velocity*

Continuous Motion – Instant. Velocity

- By definition, *instantaneous velocity*

$$v = \lim_{\Delta t \to 0}\left(\frac{\Delta s}{\Delta t}\right)$$

- Alternately represented as

$$v = \frac{ds}{dt} \qquad (1)$$

Yves Rossy. “Rocketman”

Continuous Motion – Velocity

- *Velocity* is a vector quantity
- If we had moved to the left, the velocity would be a negative value
- The magnitude of the *velocity* is called *speed*
- The units of *velocity* (and *speed*) include ft/s, mph (miles per hour), m/s, kph (kilometers per hour)

Continuous Motion – Average Speed

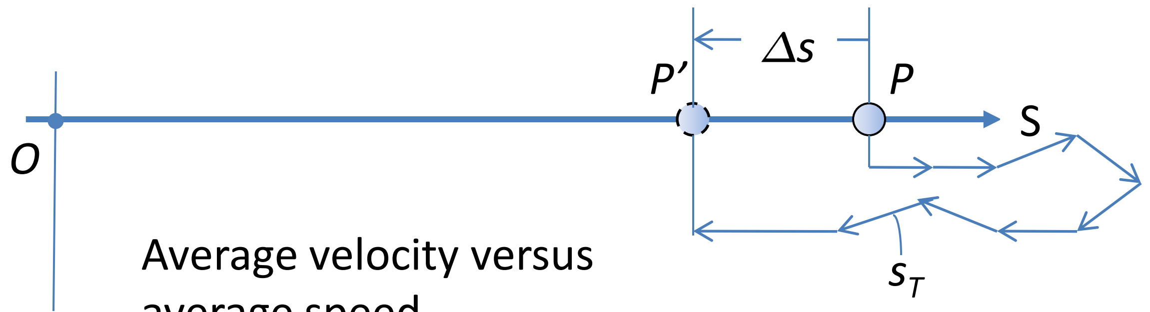
- Average speed is a positive scalar value defined as the total distance divided by the time elapsed

$$\left(v_{sp}\right)_{avg} = \frac{s_T}{\Delta t}$$

Pit Sop. McClaren Mercedes Team, Formula 1

Continuous Motion – Average Speed

- Consider the following motion that occurs over a time interval Δt



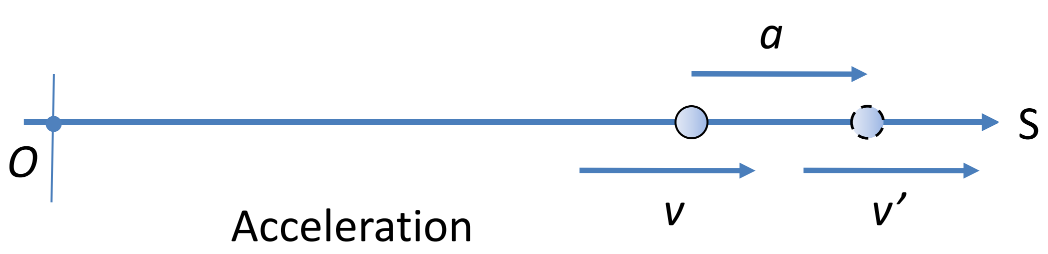
Average velocity versus average speed

- $$\left(v_{sp}\right)_{avg} = \frac{s_T}{\Delta t} \quad \text{but} \quad v_{avg} = \frac{\Delta s}{\Delta t}$$

Continuous Motion - Acceleration

- If the velocity at two instances is known then can obtain the *average acceleration* of the object during the time interval Δt as

$$a_{avg} = \frac{\Delta v}{\Delta t}$$



Continuous Motion - Acceleration

$$\Delta v = v' - v$$

- If we reduce Δt to an infinitesimally small interval (aka instant), we get the *instantaneous acceleration*

$$a = \lim_{\Delta t \to 0}\left(\frac{\Delta v}{\Delta t}\right) \quad \text{or} \quad a = \frac{dv}{dt} \qquad (2)$$

- We can see that acceleration is a vector quantity

Continuous Motion - Acceleration

- Commonly used units: ft/s^2, m/s^2, etc
- Substituting Eqn (2) in Eqn (1);

$$a = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2 s}{dt^2}$$

- From Eqn (1) and Eqn (2) if velocity is constant, then $a = 0$
- If $v' < v$, then we will have a negative value of acceleration. This is called *deceleration*

Continuous Motion - Acceleration

- From Eqn (1) we can write

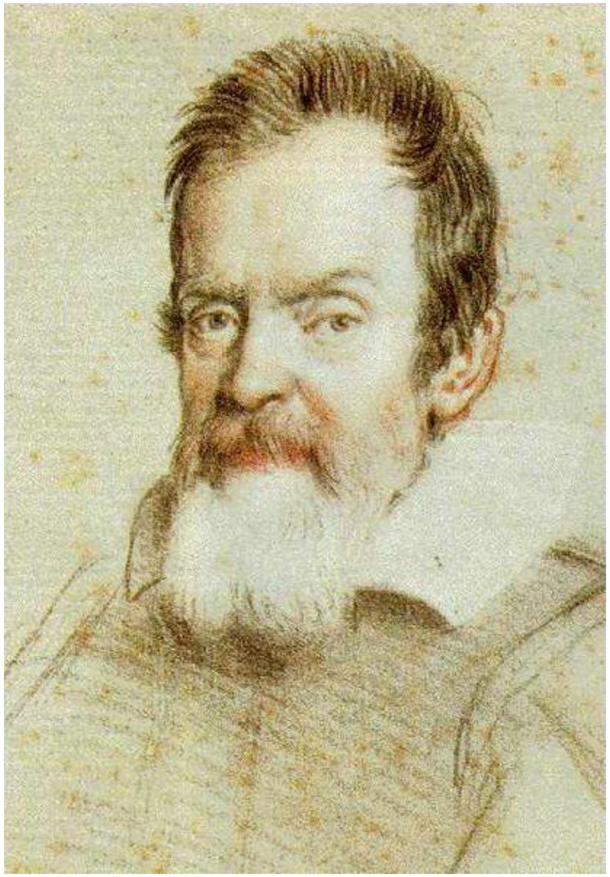
$$dt = \frac{ds}{v}$$

- From Eqn (2)

$$dt = \frac{dv}{a}$$

- Equating the above

$$a\,ds = v\,dv \qquad (3)$$



Galileo Galilei

Equations of Motion: Under Constant Acceleration

Consider the following:

- our acceleration to be constant, i.e. $a = a_c$
- at $t = 0$, $v = v_{0,}$ and $s = s_0$
- From Eqn (2)

$$a_c = \frac{dv}{dt}$$

- Rearranging and integrating

$$\int_{v_0}^{v} dv = \int_{0}^{t} a_c dt$$

Motion Under Constant Acceleration

- Solving the definite integral, we obtain velocity as a function of time:

$$\boxed{v = v_0 + a_c\, t \qquad (4)}$$

- Substituting Eqn (4) into Eqn (1)

$$v = \frac{ds}{dt} = v_0 + a_c\, t$$

$$\int_{s_0}^{s} ds = \int_0^t \left(v_0 + a_c\, t \right) dt$$

Motion Under Constant Acceleration

- we obtain position as a function of time:

$$\boxed{s = s_0 + v_0 t + \tfrac{1}{2} a_c t^2 \qquad (5)}$$

- We can rearrange Eqn (4) as

$$t = \frac{v - v_0}{a_c}$$

and substitute in Eqn (5)

Motion Under Constant Acceleration

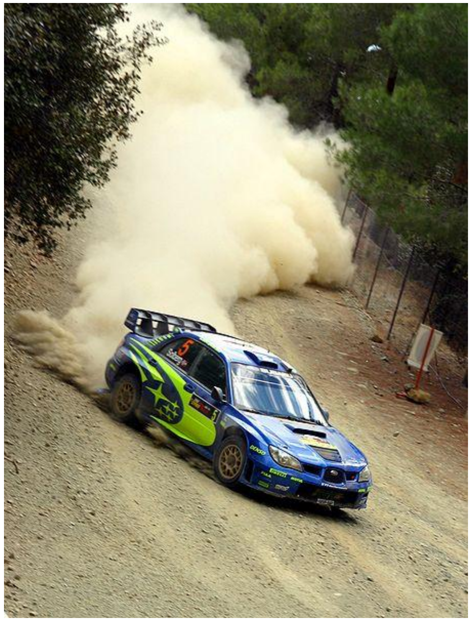
- We obtain velocity as a function of position

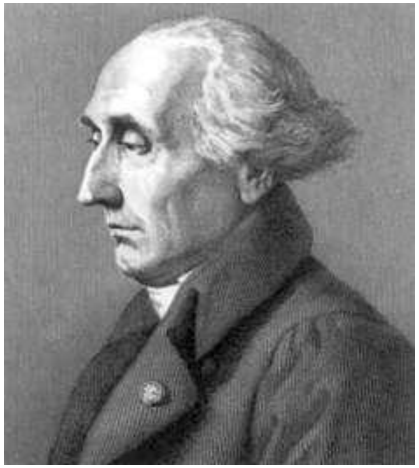
$$v^2 = v_0^2 + 2a_c\left(s - s_0\right) \qquad (6)$$

Apollo 11 Launch

How Did That Go ?

- Examples





Joseph Louis Lagrange

Rectilinear Kinematics

Erratic Motion

Overview

- Erratic Motion
- Graphical approach
- Sample problems

Robert H. Goddard (1926)
Rocket pioneer

Erratic Motion

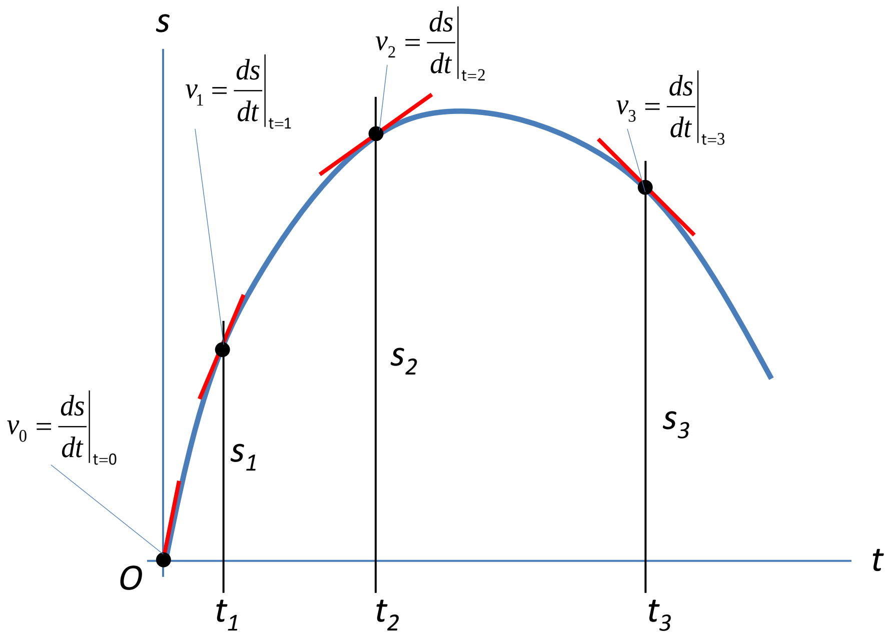
- When the motion of an object is erratic, we cannot use the single continuous function to describe its kinematics
- In other words the acceleration is not constant
- Series of functions have to be used to specify the motion over different time intervals
- In general graphs are used to facilitate the calculations

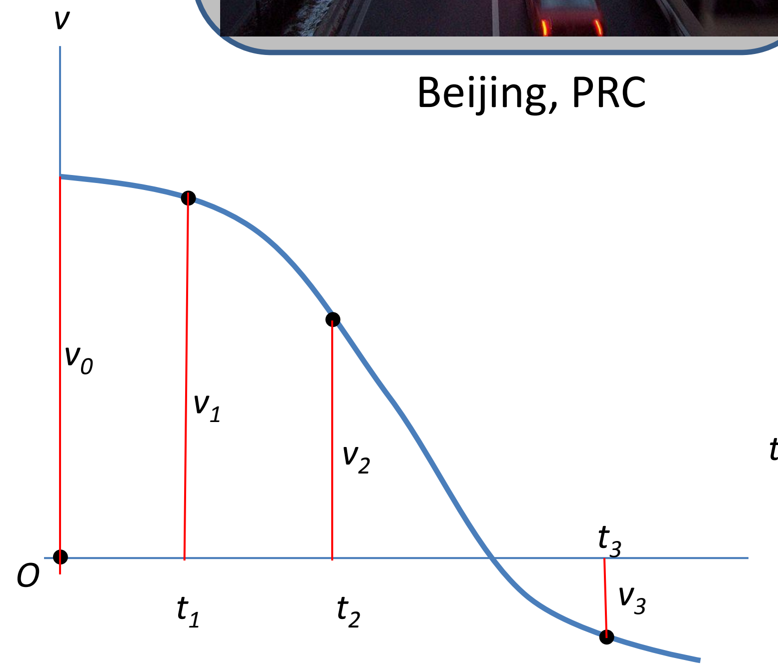
Pierre-Simon Laplace

Velocity = Slope of $s - t$ graph at time t

$$\frac{ds}{dt} = v$$

Beijing, PRC

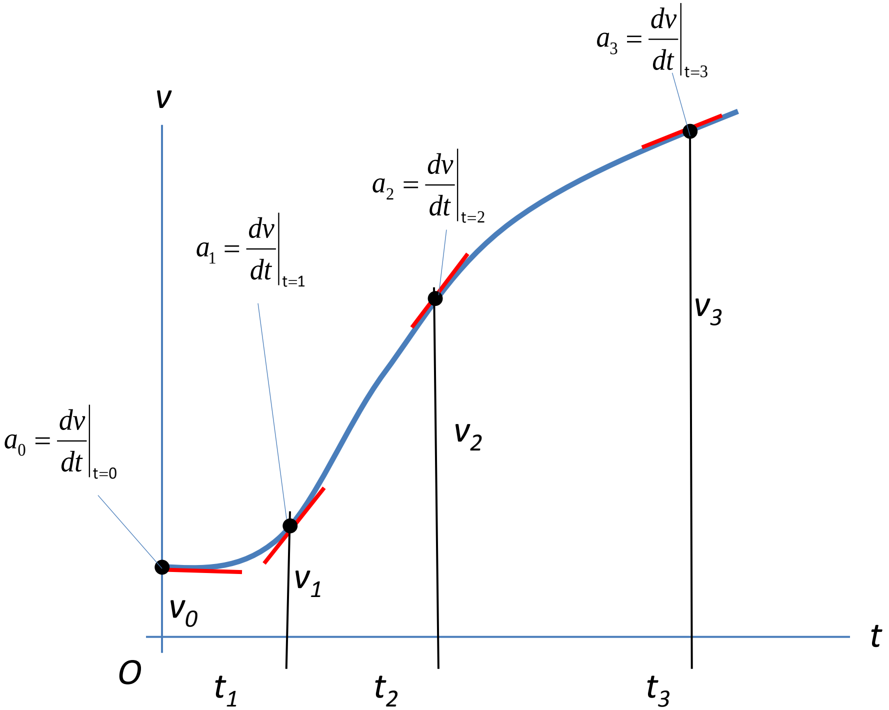


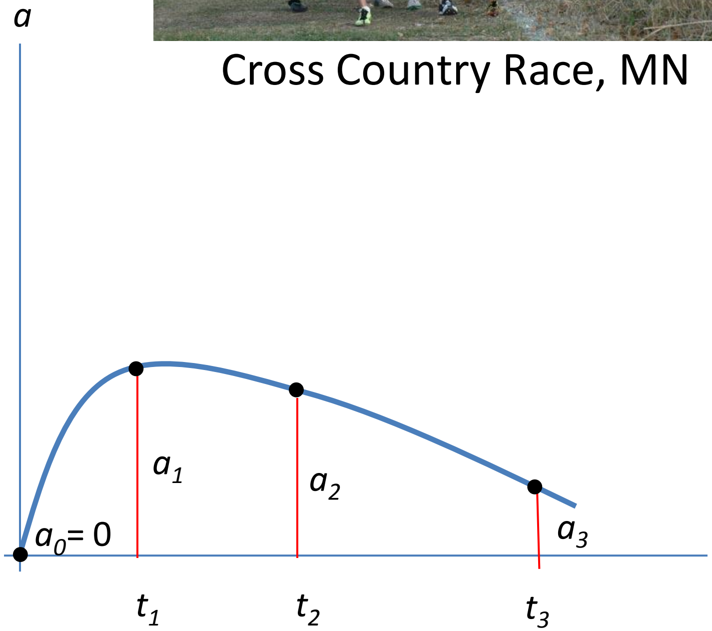


Acceleration = Slope of $v - t$ graph at time t

$$\frac{dv}{dt} = a$$

Cross Country Race, MN



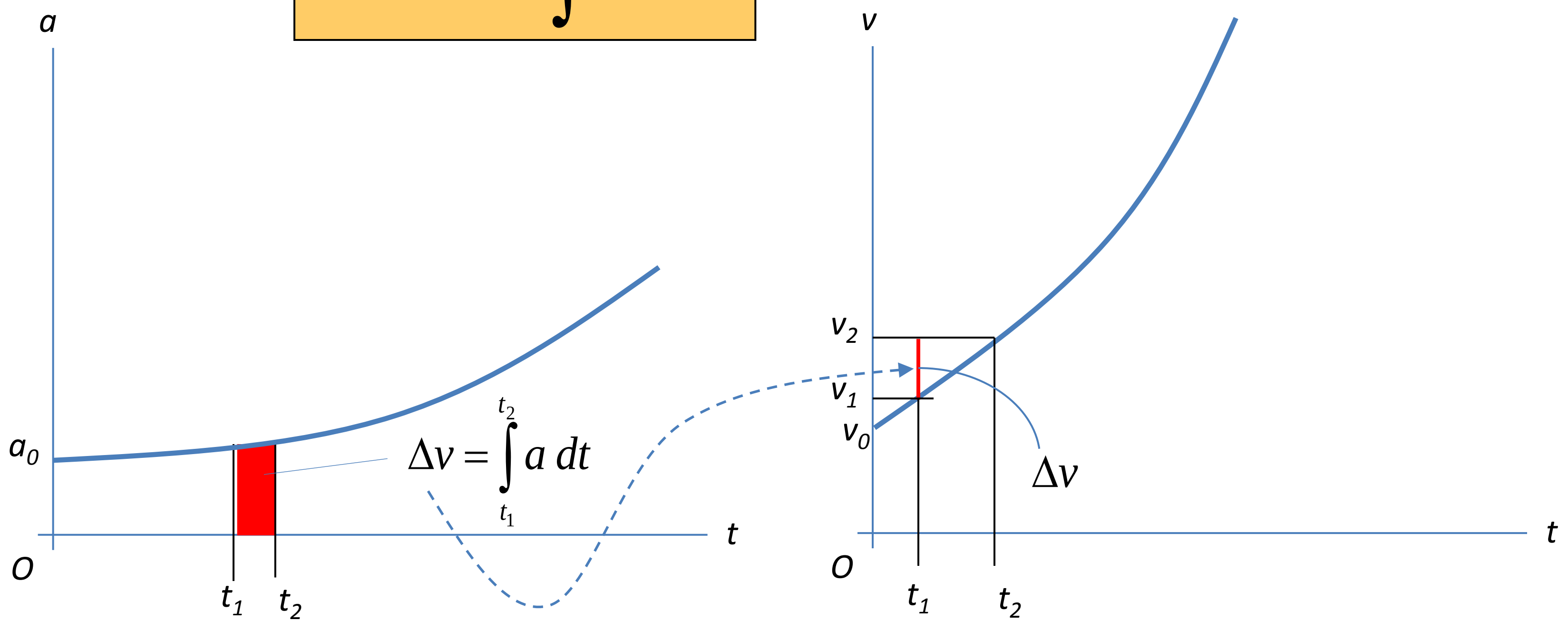


Change in velocity from $a - t$ graph

$$\frac{dv}{dt} = a$$

$$dv = a\,dt$$

$$\Delta v = \int a\,dt$$

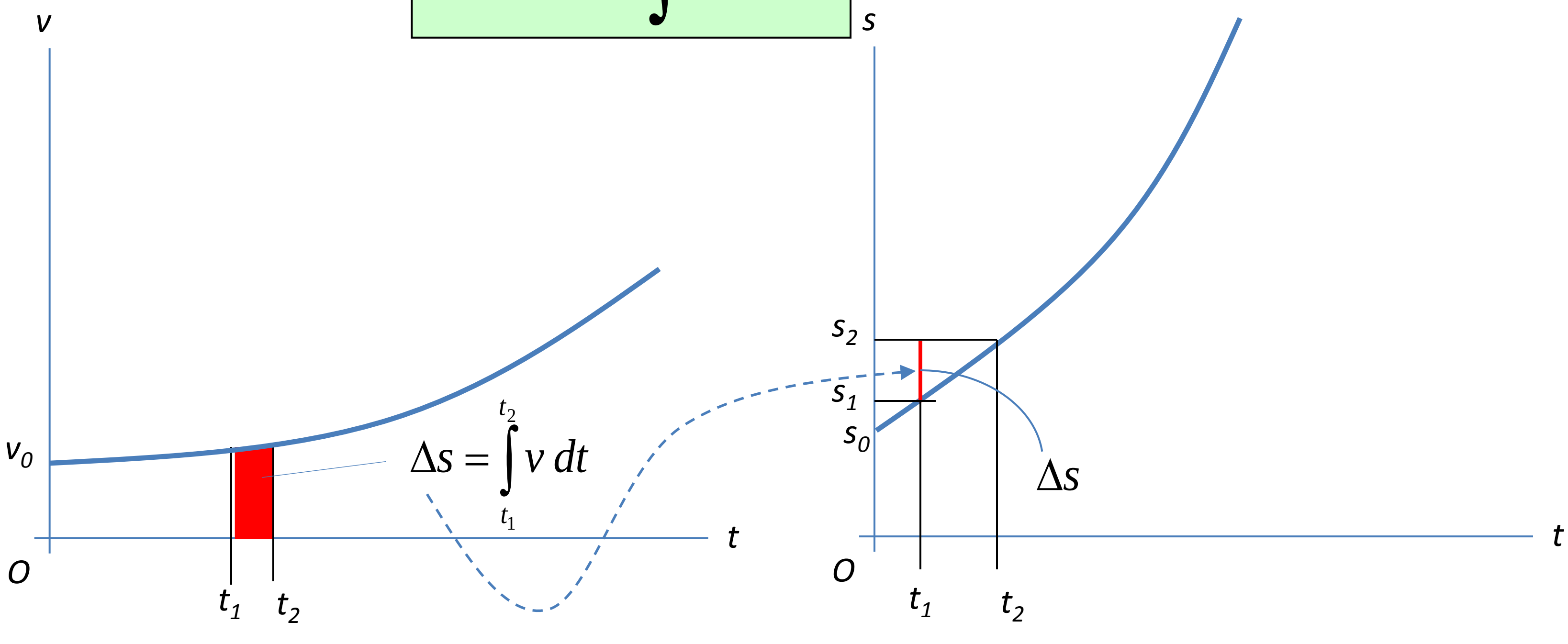


Displacement from $v - t$ graph

$$\frac{ds}{dt} = v$$

$$ds = v\,dt$$

$$\Delta s = \int v\,dt$$

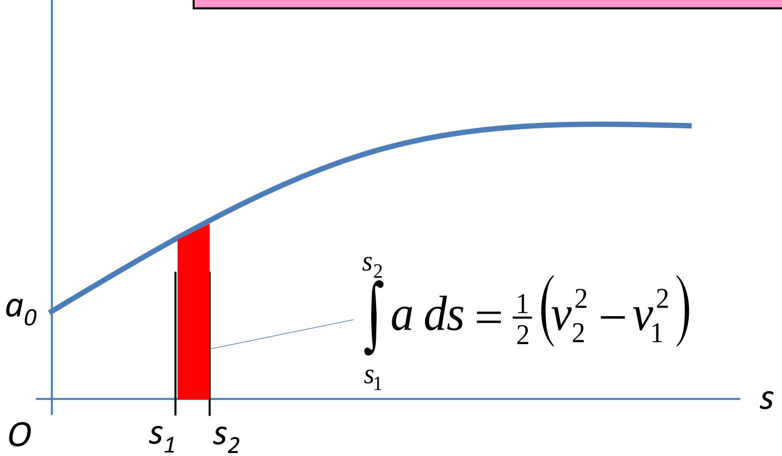


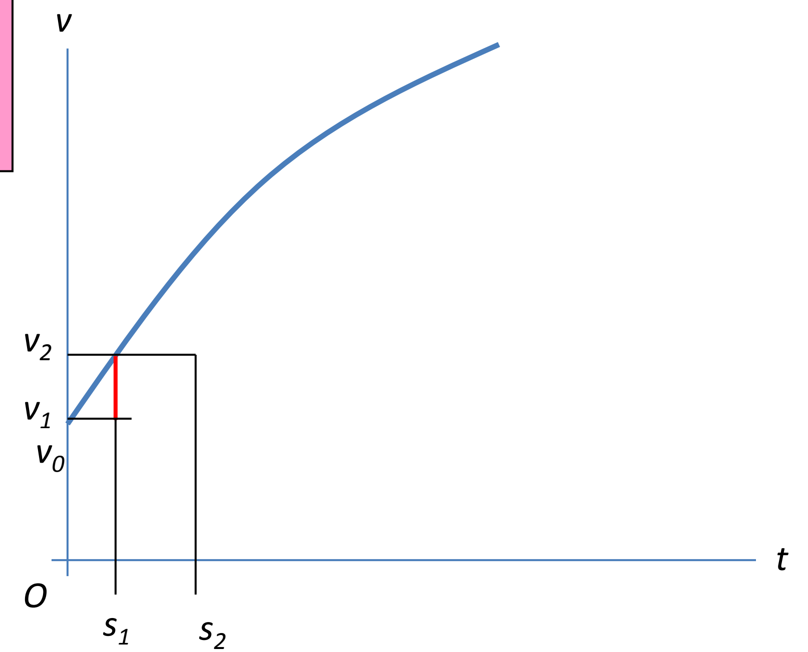
Graphs of $v - s$ & $a - s$ (Velocity)

$$a\,ds = v\,dv$$

$$\int_{t_1}^{t_2} a\,ds = \int_{v_1}^{v_2} v\,dv$$

$$\int_{t_1}^{t_2} a\,ds = \tfrac{1}{2}\left(v_2^2 - v_1^2\right)$$

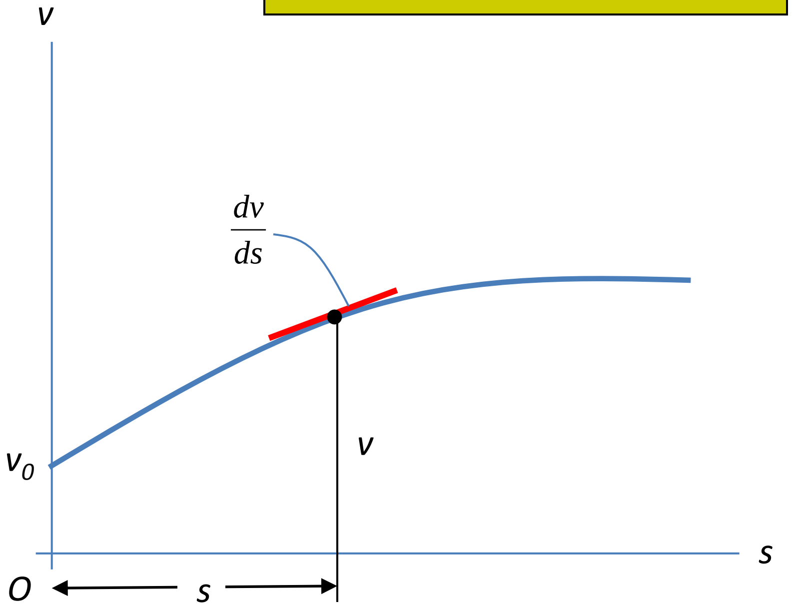


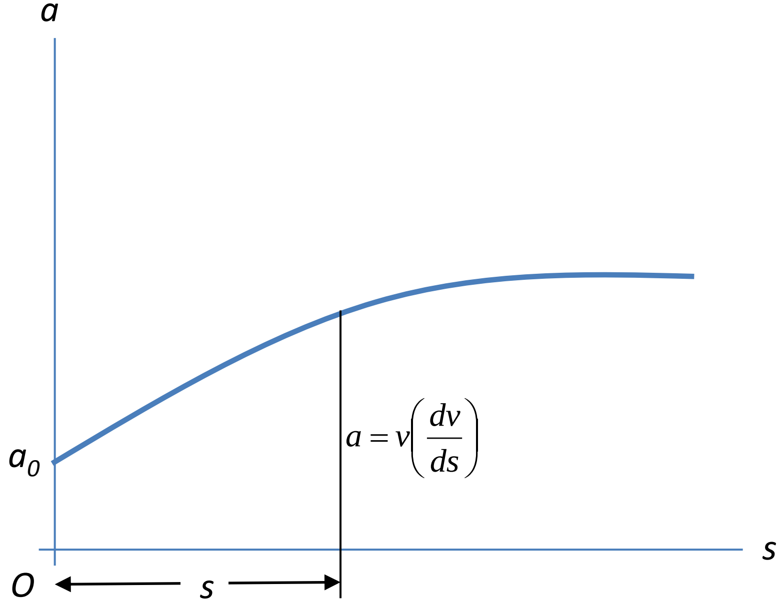


Graphs of $v - s$ & $a - s$ (Acceleration)

$$a\,ds = v\,dv$$

$$a = v\left(\frac{dv}{ds}\right)$$





How Did That Go ?

- Examples