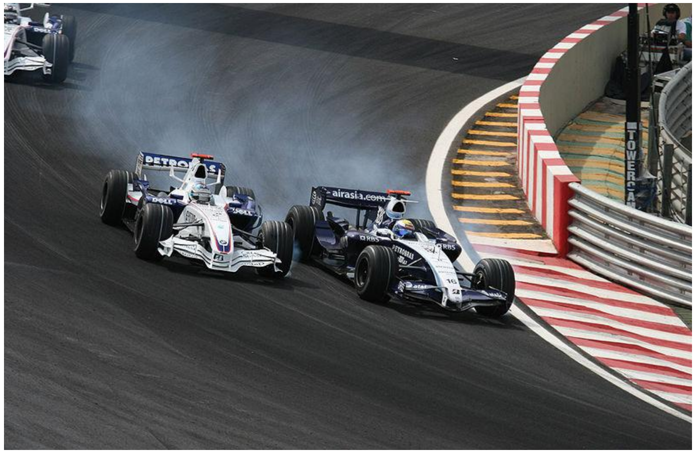
Kinematics

Absolute Dependant Motion Analysis of Two Particles
&
Relative Motion

Overview

- Absolute Dependant Motion Analysis of Two Particles
- Relative Motion

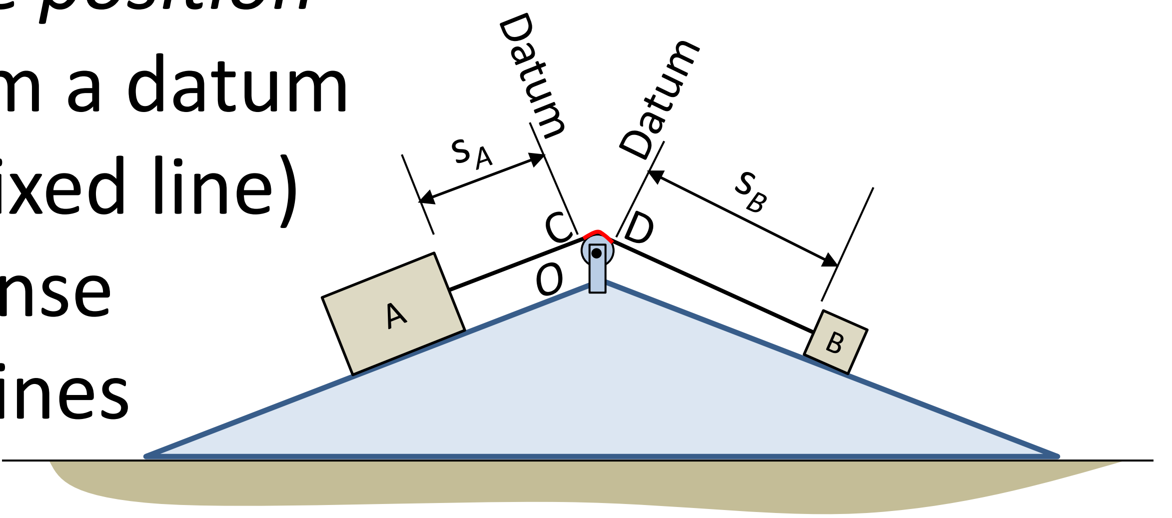



Kinematics

Absolute Dependant Motion Analysis of Two Particles

Absolute Dependent Motion

- In some cases the motion of one particle depends on the motion of another.
- Consider two objects physically interconnected by inextensible chords of a pulley system.
- s_A and s_B are the *position coordinates* from a datum (fixed point or fixed line) with positive sense "down" the inclines



Absolute Dependent Motion

- Total chord length,

$$l_T = s_A + l_{CD} + s_B$$

where l_{CD} is the chord length over arc CD of the pulley

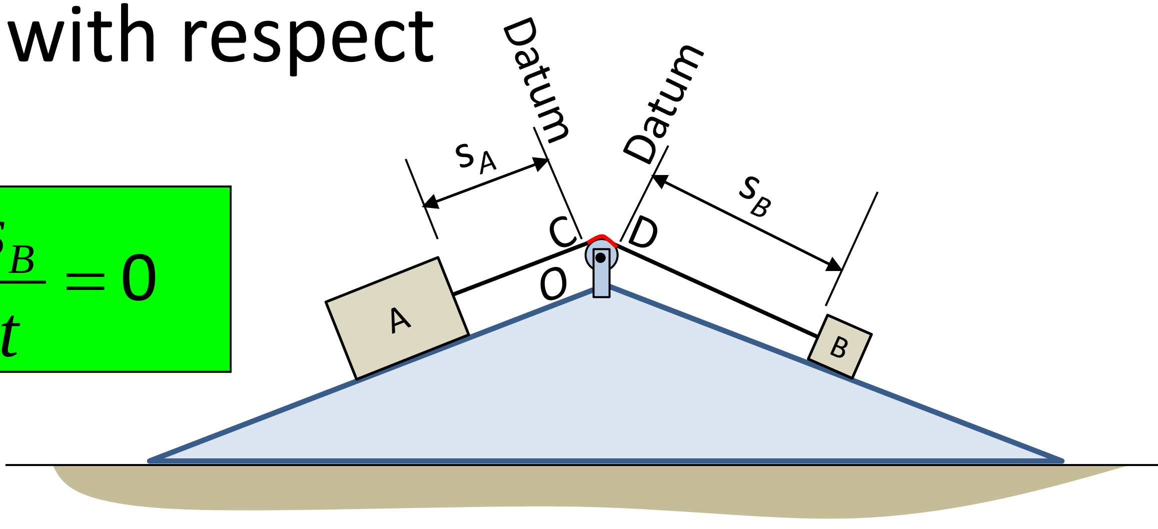
l_{CD} and l_T remain constant

- Differentiating l_T with respect to time

$$\frac{ds_A}{dt} + \frac{ds_B}{dt} = 0$$

or

$$v_B = -v_A$$



Absolute Dependent Motion

- The negative sign confirms that "upward" velocity of A causes "downward" velocity of B and vice versa.
- Differentiating velocity yields acceleration

$$\boxed{a_B = -a_A}$$

Ship elevator, Three Gorges Dam, China

Aircraft elevator, USS Kitty Hawk

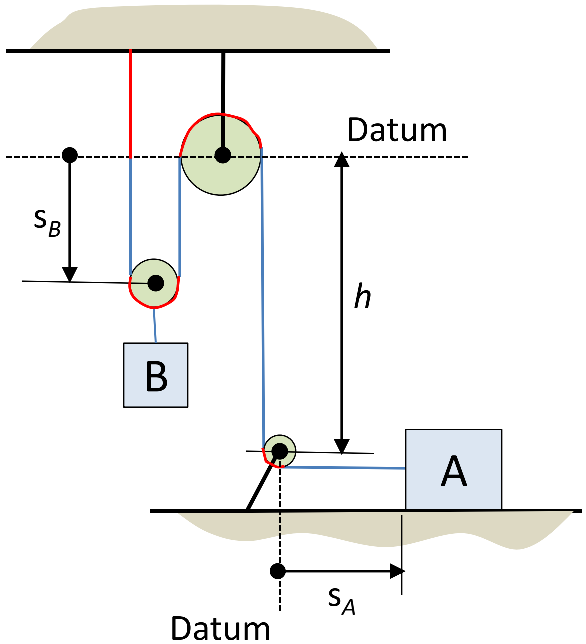
Absolute Dependent Motion

- Let us consider a more complex system
- The red colored segments of the chord will remain constant and can be omitted
- Consider positive position coordinates as shown by arrowheads
- For total length of the chord, *l*

$$\boxed{2s_B + h + s_A = l}$$

in this set up *h* is constant, therefore time derivatives are

$$\boxed{2v_B = -v_A} \qquad \boxed{2a_B = -a_A}$$



Absolute Dependent Motion

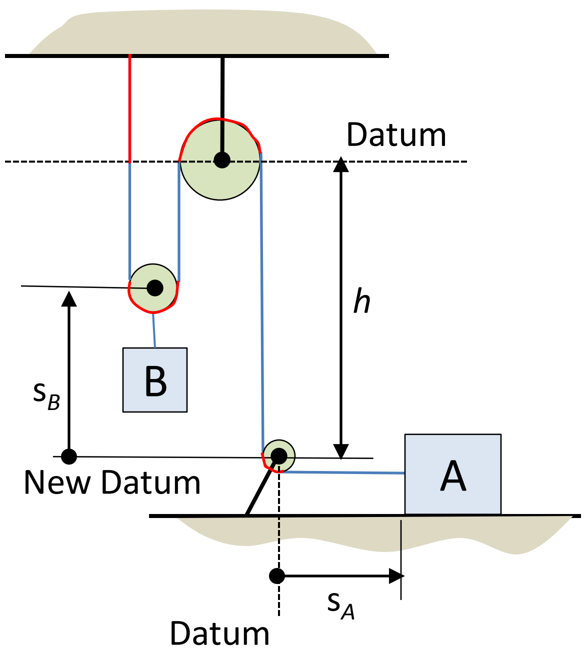
- We could have defined the position of object B from the small pulley at the bottom of the assembly
- total length of the chord, *l*

$$2(h - s_B) + h + s_A = l$$

therefore time derivatives are

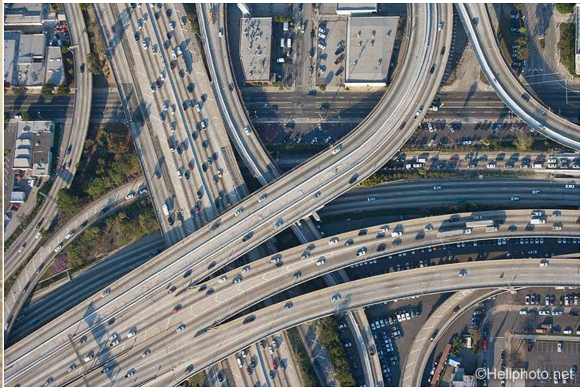
$$2v_B = v_A$$

$$2a_B = a_A$$



Conclusion

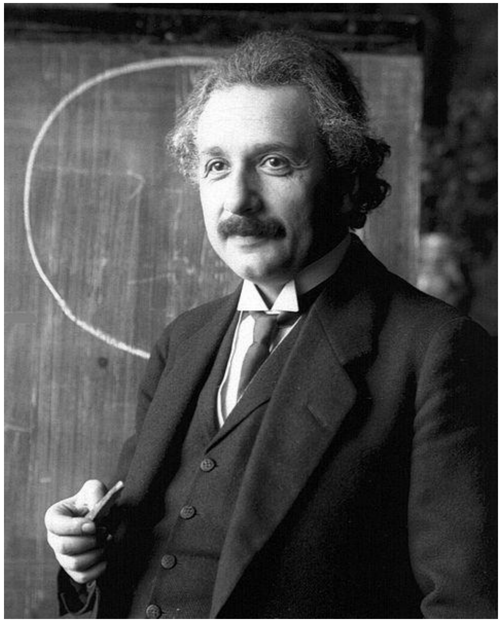
- Examples



Apollo-Soyuz Rendezvous

Kinematics

Relative Motion



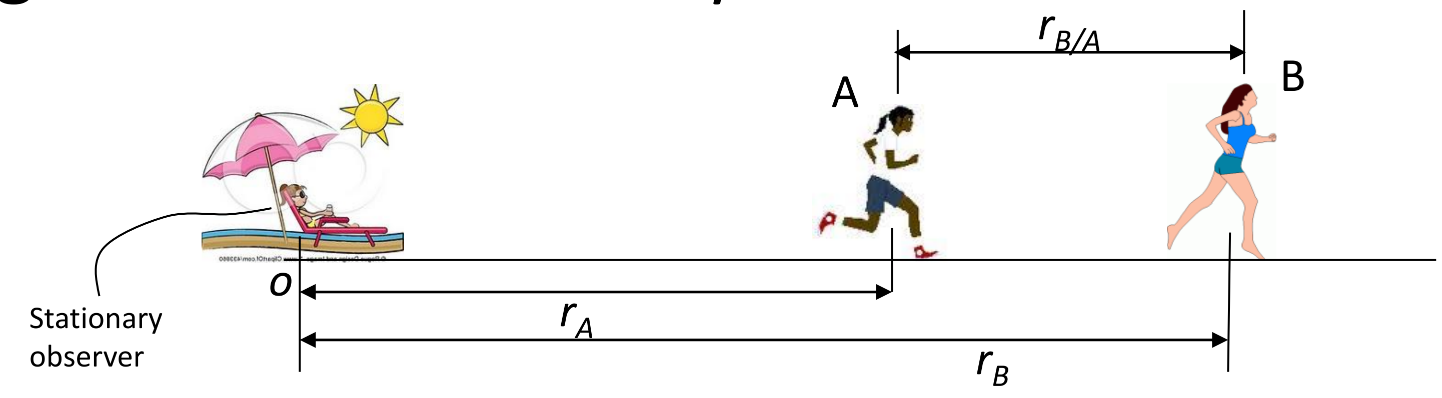
Albert Einstein

Relative Motion-Translating Axes

- So far we have considered absolute motion of a particle within a fixed frame of reference
- In some cases it is easier to analyze the motion using two or more frames of reference
- In this chapter we shall set up another frame of reference by *translating frames of reference* to change our point of view of the object(s) in motion.

Relative Motion

- Consider two joggers (A and B) on the beach running in a straight path and being monitored by a stationary observer at a fixed origin *O*.
- At an instant, with this frame of reference, we can consider the positions of the joggers from the origin as their *absolute position*



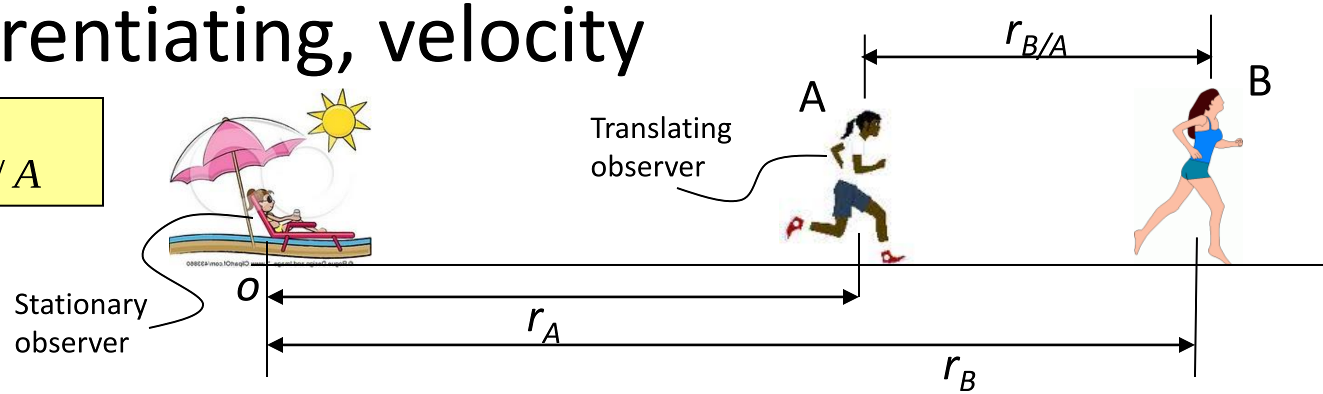
Relative Position

- However, if we were to change our perspective and consider the position of B as observed by A, we would call this the *position of B relative to A*, or the *relative position of B from A,* and denote it by $r_{B/A}$
- It follows that

$$\boxed{r_{B/A} = r_B - r_A} \quad \text{or} \quad \boxed{r_B = r_A + r_{B/A}}$$

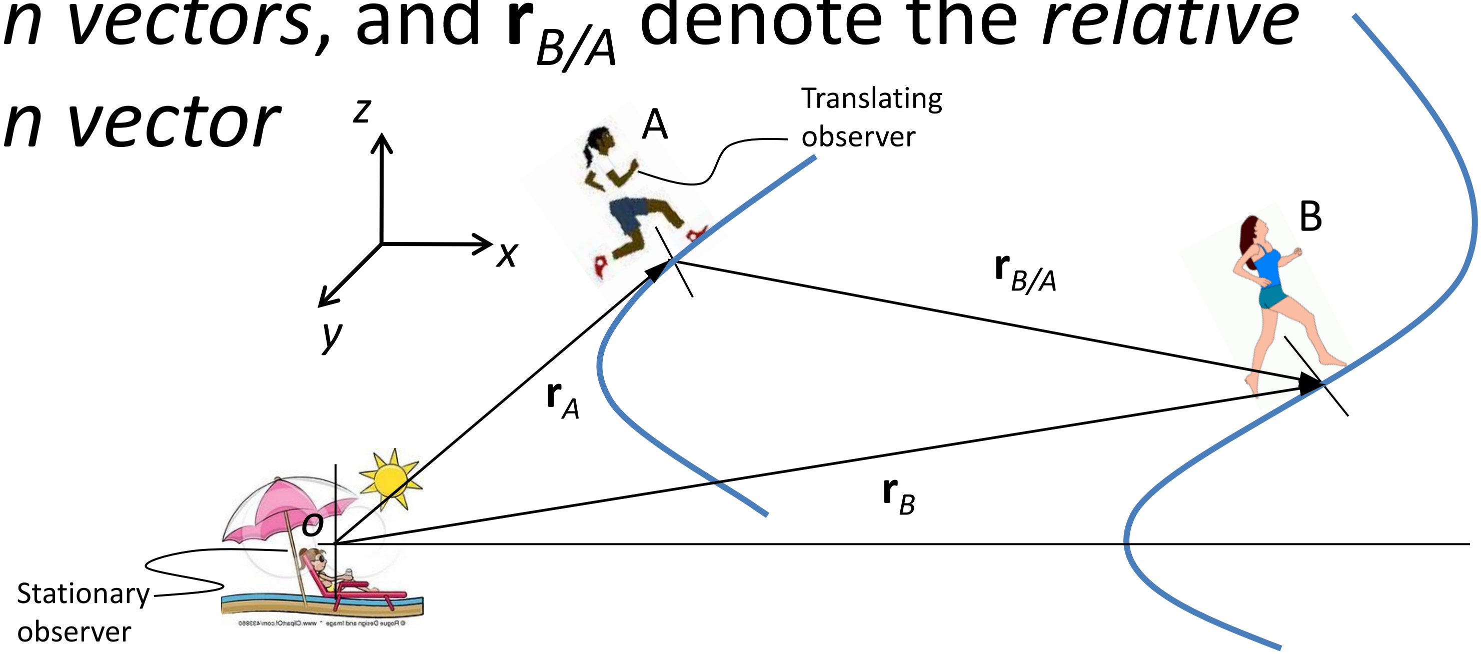
and by differentiating, velocity

$$\boxed{v_B = v_A + v_{B/A}}$$



Translating Axes

- We can extend this concept to vectors in order to analyze motion along curvilinear paths that also may not coincide.
- Let $\mathbf{r}_A$, $\mathbf{r}_B$, denote the corresponding *absolute position vectors*, and $\mathbf{r}_{B/A}$ denote the *relative position vector*



Then:

Relative Motion-Translating Axes

- Position:

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$$

- The time derivative of position yields the *absolute and relative velocity*;

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

$\mathbf{v}_{B/A}$ is the velocity of jogger B as observed by jogger A who is also in motion with absolute velocity $\mathbf{v}_A$

- The time derivative of velocity yields the acceleration;

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$

Conclusion

- Examples