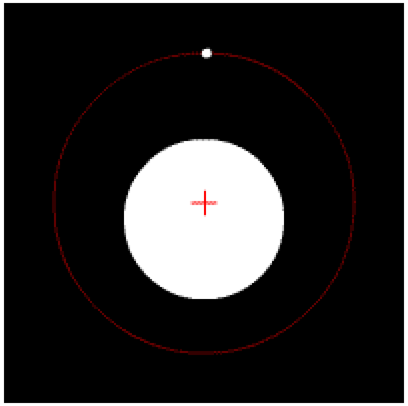
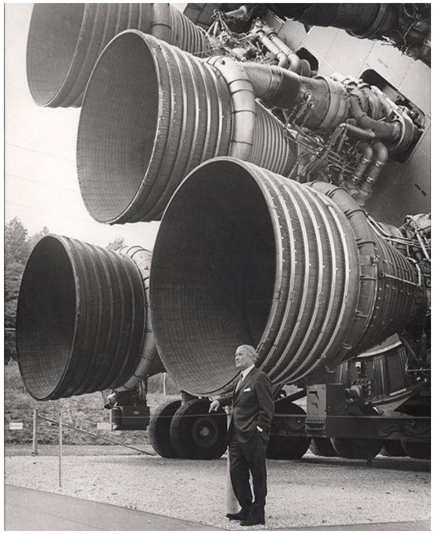
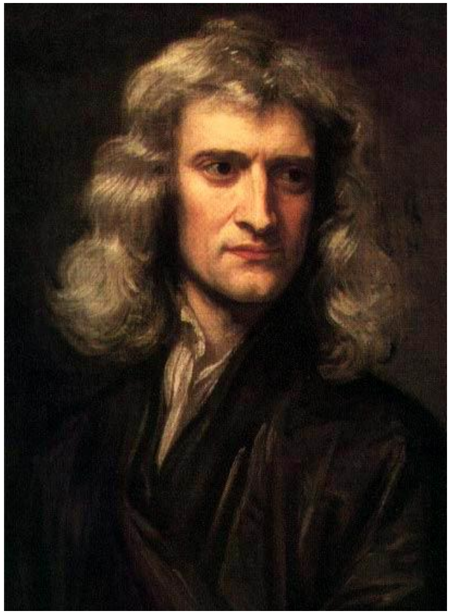




Kinetics of a Particle



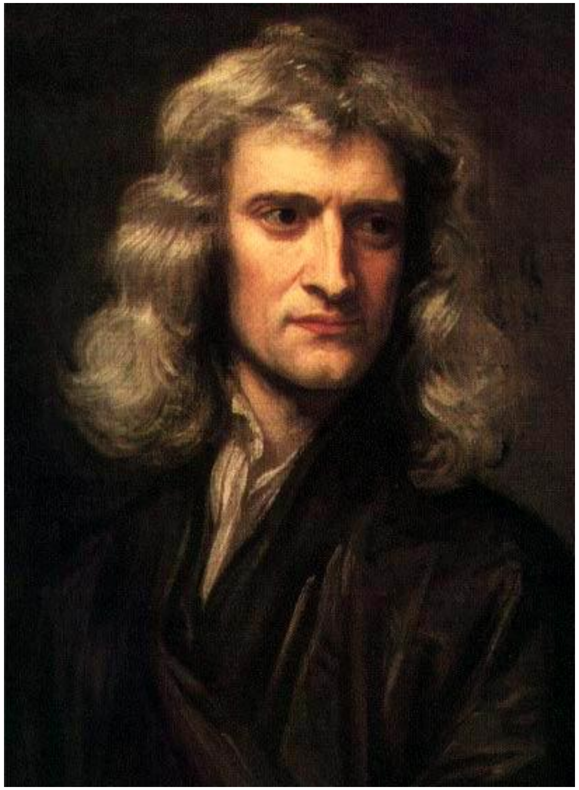
Force and Acceleration

Overview

- Kinetics
- Newton’s Second Law of Motion
- The Equation of Motion
- Equation of Motion: System of Particles
- Equations of Motion: Rectangular Coordinates
- Equations of Motion: Normal and Tangential Coordinates

Kinetics

- The branch of dynamics that deals with the relationship between the change in motion of an object and the forces that cause this change.
- The basis for kinetics is form Newton’s second law (1686) which states that when an *unbalanced force* acts on a particle, the particle will *accelerate* in the direction of the force with a magnitude proportional to the force.



Sir Isaac Newton

Newton's 2nd Law

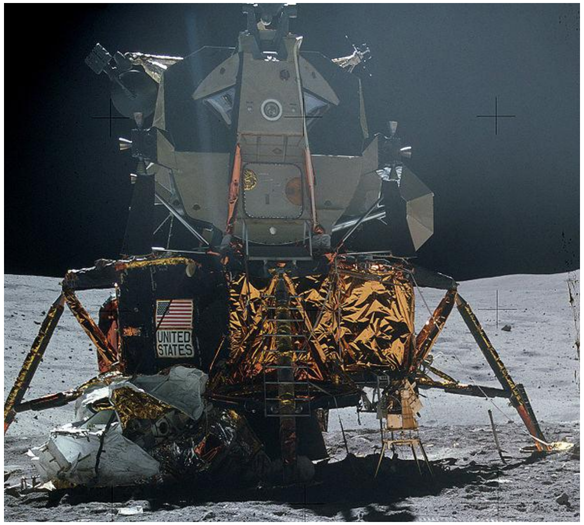
$$\mathbf{F} \propto \mathbf{a}$$

$$\mathbf{F} = m\mathbf{a}$$

- *m* is the *mass* of the object.
- Mass is the quantity of matter an object has, it is a scalar.
- The mass provides the resistance of the object to any change in its velocity, that is its inertia

Newton's 2nd Law

- Arguably one of the most important equations in modern physics.
- A cornerstone of our modern built environment, and engineering
- Validity based solely on experimental evidence.



Newton's 2^{nd} Law – Disclaimer !

- Albert Einstein (1905) showed that time is relative, as a result Newton's 2^{nd} law fails to accurately describe the motion of objects traveling near the speed of light.
- Advances in the Quantum physics have shown that motion of atoms and subatomic particles do not obey Newton's 2^{nd} law
- For most problems in modern engineering however neither of the above are applicable

Newton's Law of Universal Gravitation

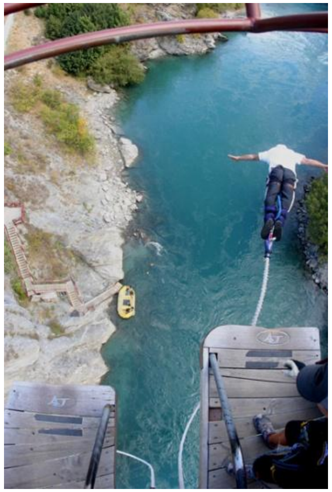
- Newton (1687) postulated a law governing the mutual attraction between any two objects in the universe

$$F = G\frac{m_1 m_2}{r^2}$$

where

F = force of attraction between two particles

G = universal constant of gravitation, $33.73(10^{-12})\,m^3/(kg.s^2)$



m_1, m_2 = mass of each particle respectively

r = distance between the centers of the two particles

Law of Gravitational Attraction

- For an object "near" the earth surface we call this force of attraction between the object and the earth the *weight* (*W*) of the object
- If $m_1 = m$, and M_e is the mass of the earth,
- $$F = G\frac{M_e m}{r^2}$$

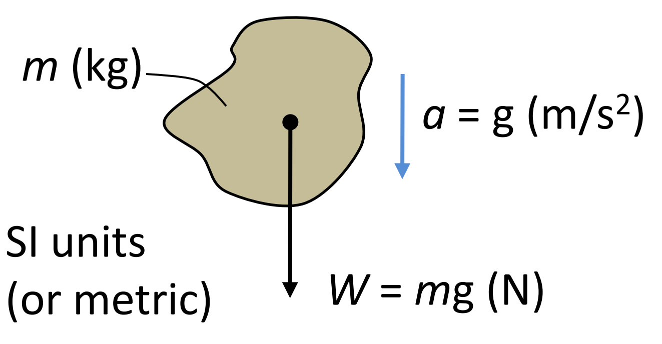
$$\boxed{W = mg}$$

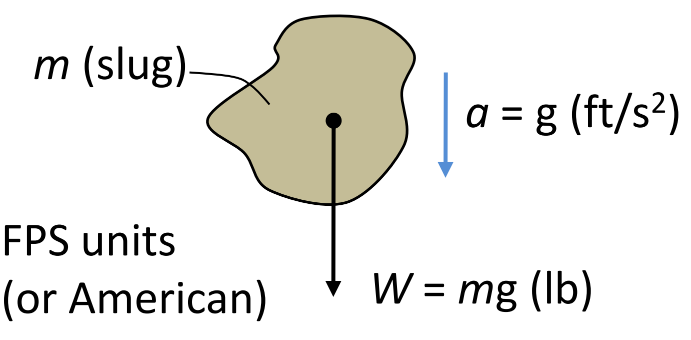
where

$$g = \frac{GM_e}{r^2}$$

Acceleration Due to Gravity

- g is called the *acceleration due to gravity*
- In engineering calculations g = 9.81 m/s^2 or 32.2ft/s^2
- Units of weight are *Newtons* (N) and *pounds* (lb)
- Units of mass are *kilograms* (kg) and *slug*

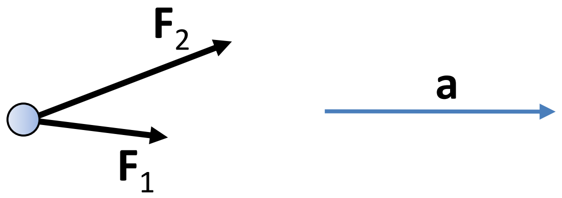




The Equation of Motion

- When more than one force acts on an object, the *resultant force* is vector summation of all the forces

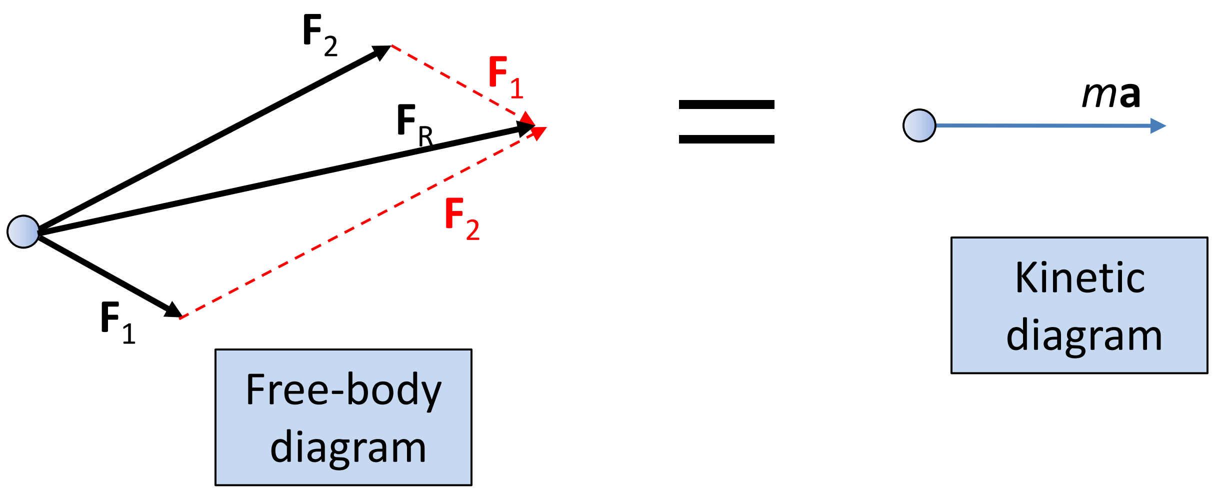
$$\mathbf{F}_R = \sum_i \mathbf{F}_i$$



$$\boxed{\sum_i \mathbf{F}_i = m\mathbf{a}}$$

Equation of Motion

- Consider an object of mass m subject to the action of two forces F_1 and F_2. We can draw a graphical representation of the resultant force as follows:

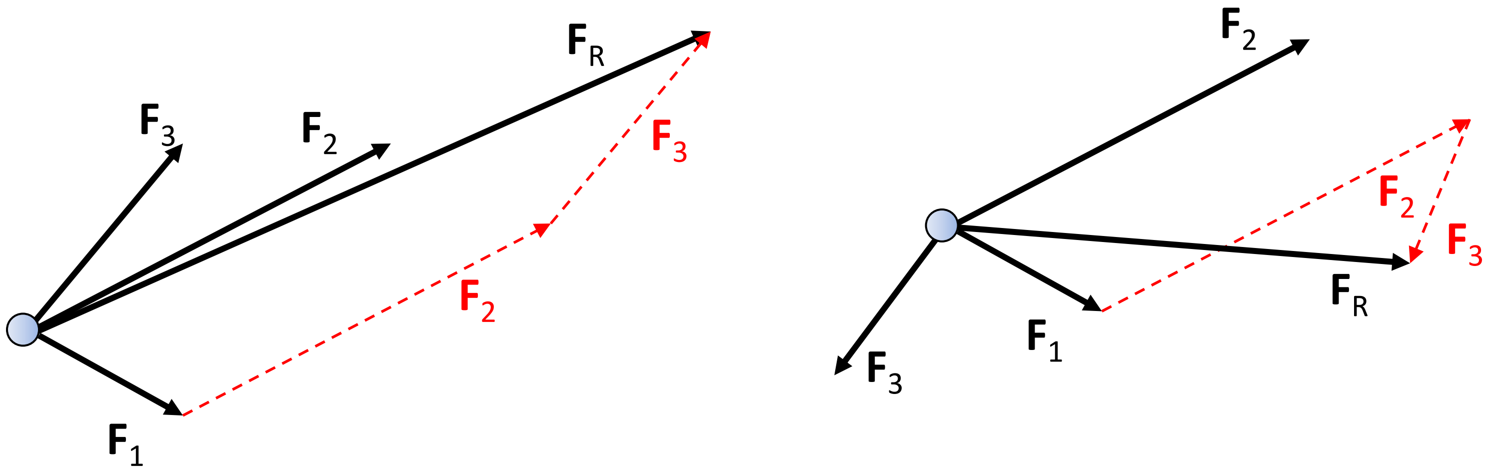


Free-body Diagram

- Note that if F_R is zero, the acceleration is also zero. This means the object will either
  - Remain or rest
  - Have constant velocity
- Remember Newton's 1st law of motion?
- We say that the free-body diagram is graphically equivalent to the kinetic diagram
- Always note the directions of your force vectors when setting up the free body diagram

Free-body Diagrams

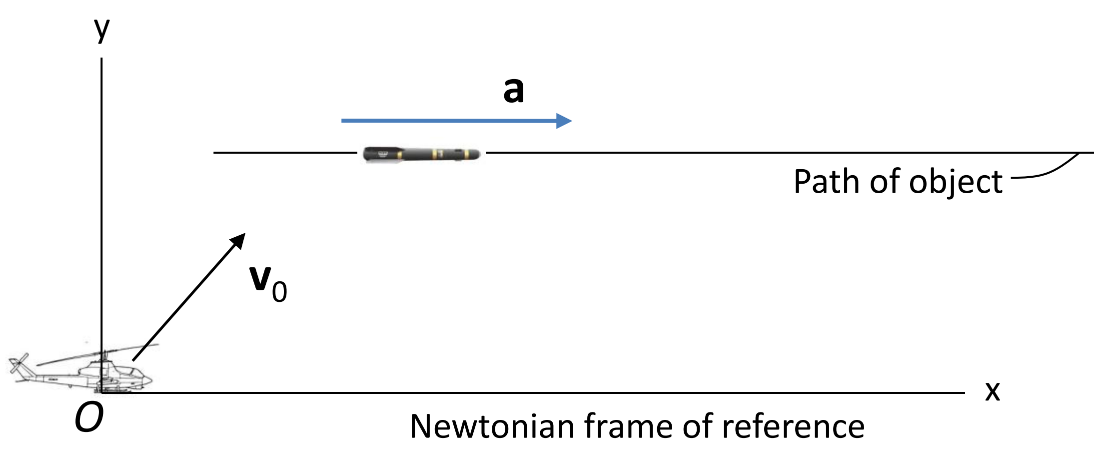
- And yes, it can get more complicated



- What if six forces act in 3-D ? Uh oh, more agony
- So in such cases visualization may become difficult, so stick with the math (force vectors summation)

Inertial Reference Frame

- To facilitate our calculations, it is important to establish a frame of reference (aka a point of view – remember relative motion?) for the acceleration **a**
- The frame of reference may be fixed or may move at a constant velocity



Inertial reference Frame

- Such a frame of reference is called a *Newtonian* or *inertial reference frame*
- For example in analyzing the motion of a aircraft it is common to use a fixed point on earth surface as the Newtonian reference frame. In astronomy, the position of known catalogued stars may be used.

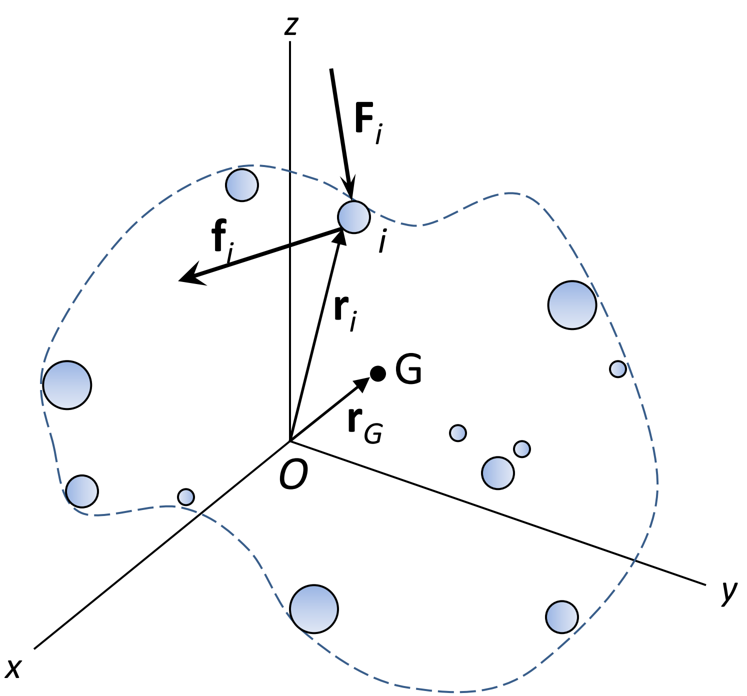
Questions & Comments

Equation of Motion for a System of Particles

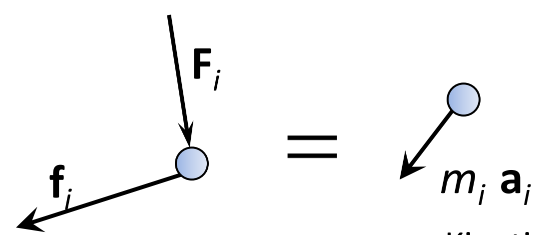
- Let us consider the motion of an isolated group of particles within a specific region of space
  Let $\mathbf{f}_i$ denote the resultant of forces all other particles exert on an arbitrary particle *i*
- Let $\mathbf{F}_i$ denote the resultant of all external forces on the system, exerted on particle *i*
- In practice $\mathbf{f}_i$ and $\mathbf{F}_i$ could represent gravitational interaction, electrostatic or magnetic forces, contact forces etc etc

Motion of a System of Particles

- At an instant, consider the *i*th particle of mass m_i, of a system of particles subject to resultant internal force $\mathbf{f}_i$ and resultant external force $\mathbf{F}_i$



Inertial coordinate system



Free-body diagram

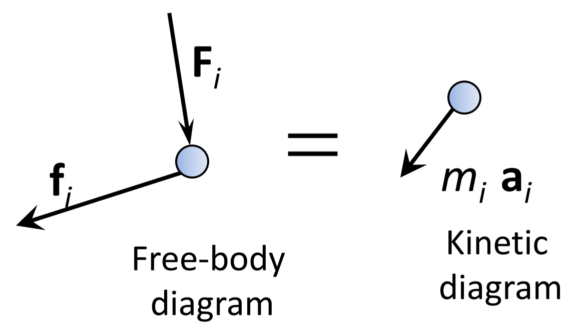
Kinetic diagram

Motion of a System of Particles

- From the free body diagram and the kinetic diagram

$$\sum \mathbf{F} = m\mathbf{a}$$

$$\mathbf{F}_i + \mathbf{f}_i = m_i \mathbf{a}_i$$



- So if we repeat for all the other particles and sum up the corresponding vectors

$$\sum_i \mathbf{F}_i + \sum_i \mathbf{f}_i = \sum_i m_i \mathbf{a}_i$$

Motion of a System of Particles

- The internal forces will cancel each other out and add up to zero, so we shall be left with

$$\sum_i \mathbf{F}_i = \sum_i m_i \mathbf{a}_i \qquad (13\text{-}5)$$

- $\mathbf{r}_G$ is the position of the center of mass (center of gravity) of the of the system of particles, so

$$m\mathbf{r}_G = \sum_i m_i \mathbf{r}_i$$

where $m = \sum_i m_i$ is the total mass of all particles

Motion of a System of Particles

- If we take the second derivative of position with respect to time we get acceleration, so

$$m\mathbf{a}_G = \sum_i m_i \mathbf{a}_i$$

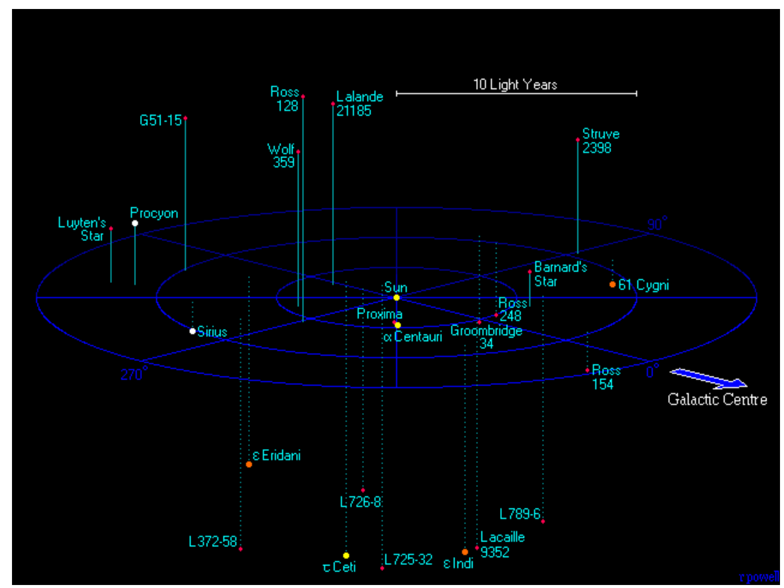
substituting into Eqn 13-5,

$$\boxed{\sum \mathbf{F} = m\mathbf{a}_G}$$

Motion of a System of Particles

- So the sum of external forces acting on the system equals the total mass of the system times the acceleration of its center of gravity
- Note that to apply this method we must always stay within our defined inertial frame of reference
- Since in reality an object is a collection of several particles, we are justified to apply this equation to a full blown object (like a planet, a car etc)
- Therefore these equations can be applied equally to liquids and gases

Questions ?


10 Light Years
Ross 128
Lalande 21185
G51-15
Wolf 359
Struve 2398
Luyten's Star
Procyon
90°
Barnard's Star
61 Cygni
Sun
Ross 248
Proxima
Sirius
α Centauri
Groombridge 34
270°
Ross 154
0°
Galactic Centre
ε Eridani
L726-8
L789-6
L372-58
Lacaille 9352
τ Ceti
L725-32
ε Indi

Equations of Motion: Rectangular Coordinates

- A particle in motion can be described by the **i**, **j**, **k** components within an inertial frame of reference
- From the equation of motion:

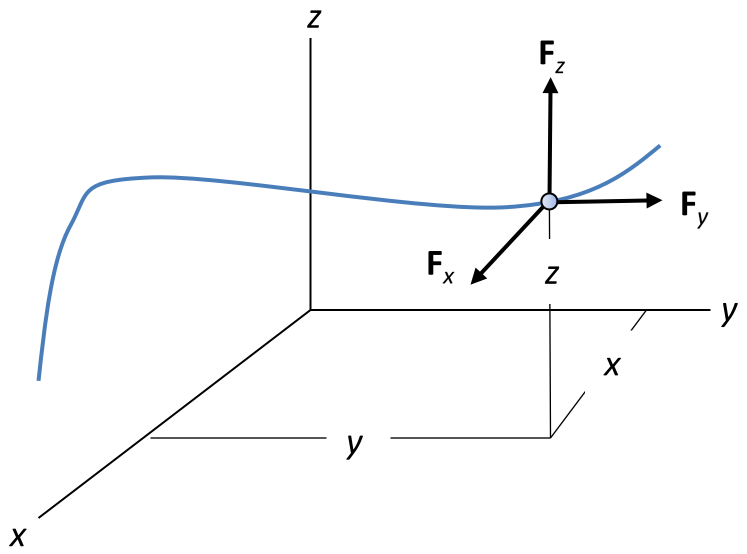
$$\sum \mathbf{F} = m\mathbf{a} \Rightarrow \sum F_x \mathbf{i} + \sum F_y \mathbf{j} + \sum F_z \mathbf{k} = m(a_x \mathbf{i} + a_y \mathbf{j} + a_z \mathbf{k})$$

where

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

$$\sum F_z = ma_z$$



Equation of Motion: Normal and Tangential Components

- If a particle moves along a known curved path, we may write the equation of motion using the tangential, normal and binormal components

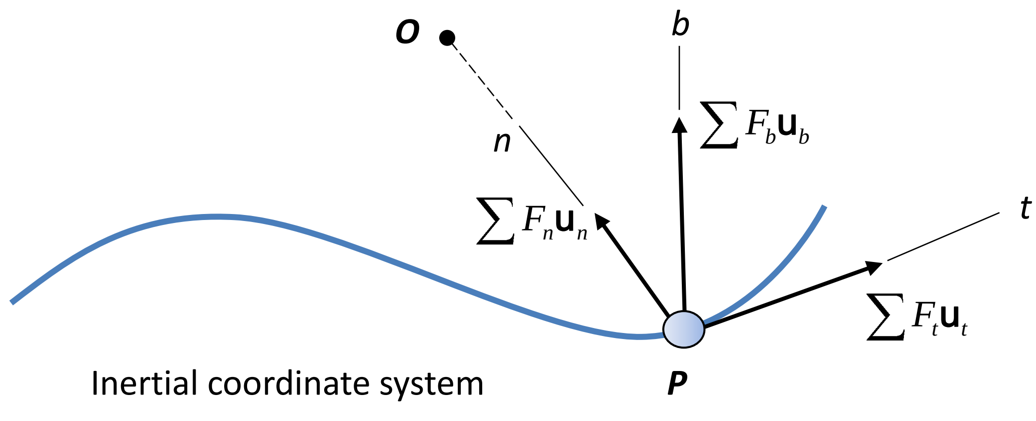
$$\sum \mathbf{F} = m\mathbf{a}$$

If the motion is constrained in a 2-d plane, the binormal component is zero

$$\sum F_t \mathbf{u}_t + \sum F_n \mathbf{u}_n + \sum F_b \mathbf{u}_b = m\mathbf{a}_t + m\mathbf{a}_n$$

Which implies

$$\boxed{\begin{aligned} \sum \mathbf{F}_t &= ma_t \\ \sum \mathbf{F}_n &= ma_n \\ \sum \mathbf{F}_b &= 0 \end{aligned}}$$



Equation of Motion: Normal and Tangential Components

- Note that if the tangential component acts in the direction of motion, the particle's speed with increase, and the converse
- The normal component of acceleration represents the rate change on the velocity’s direction and is positive towards the center of motion
- The normal force is called the *centripetal force*

Questions ?

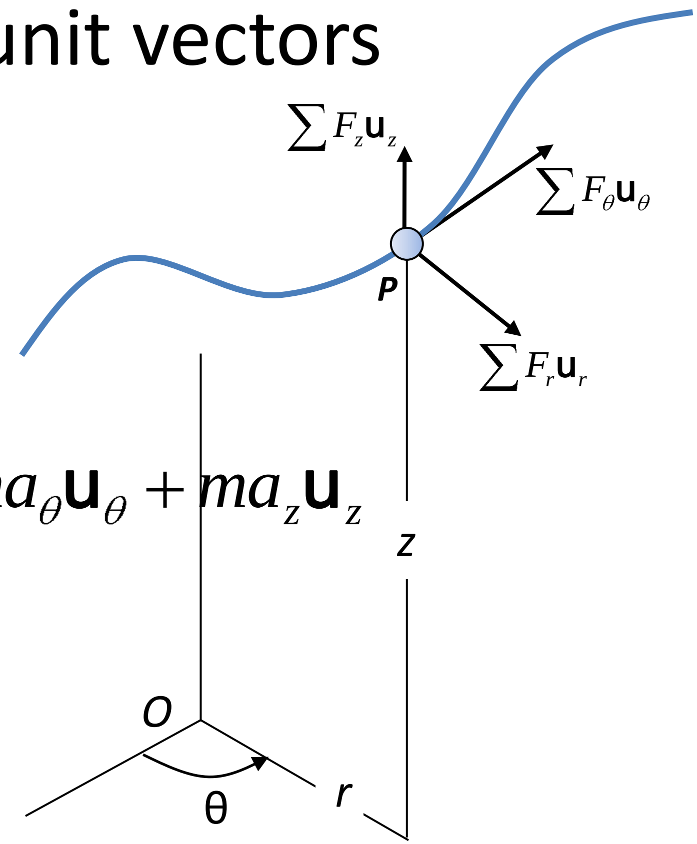
- Normal and Tangential Coordinates

Equations of Motion Cylindrical Coordinates

- The motion of a particle can be resolved into cylindrical components with unit vectors $\mathbf{u}_r$, $\mathbf{u}_\theta$, $\mathbf{u}_z$
- By the equation of motion

$$\sum \mathbf{F} = m\mathbf{a}$$

$$\sum F_r \mathbf{u}_r + \sum F_\theta \mathbf{u}_\theta + \sum F_z \mathbf{u}_z = ma_r \mathbf{u}_r + ma_\theta \mathbf{u}_\theta + ma_z \mathbf{u}_z$$

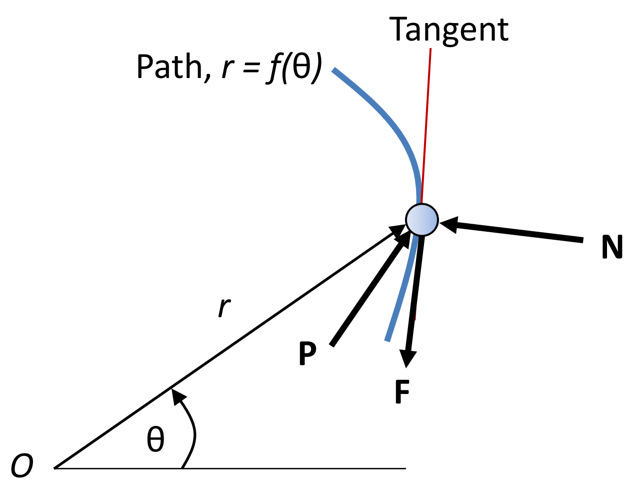


Which implies;

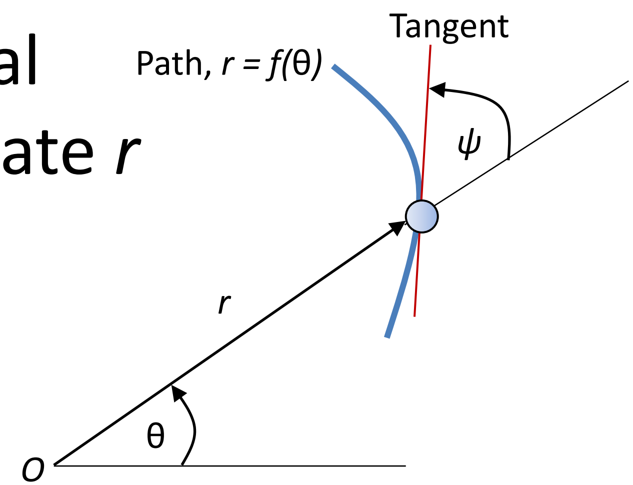
$$\boxed{\begin{aligned} \sum \mathbf{F}_r &= ma_r \\ \sum \mathbf{F}_\theta &= ma_\theta \\ \sum \mathbf{F}_z &= ma_z \end{aligned}}$$

Cylindrical Coordinates

- Consider a particle moving due a force **P** acting on it. The frictional force countering the motion will act tangentially to the path *r*, whereas a *normal force* **N** will be *perpendicular to the tangent of the path*

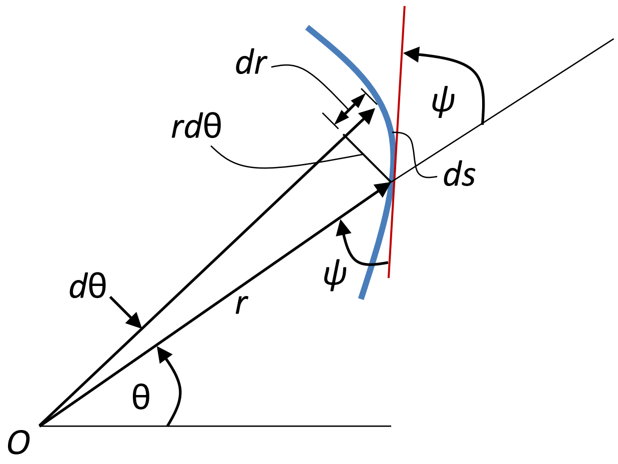


- It may be necessary to specify **N** and **F** relative to the radial coordinate *r*



Cylindrical Coordinates

- To determine ψ, we consider an infinitesimally small displacement *ds* along the path associated with a radial displacement *dr*



- *Since dr and rdθ are mutually perpendicular*

$$\tan\psi = rd\theta / dr$$

$$\tan\psi = \frac{r}{dr/d\theta}$$

Questions & Comments