Kinetics

Work & Energy

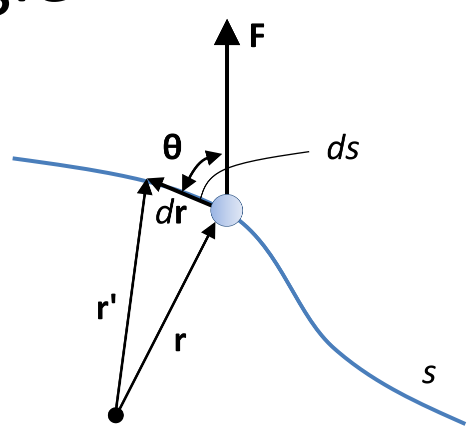
Outline

- Define Work and Energy
- Define Power and Efficiency
- Theorem of Conservation of Energy

The Work of a Force

- A force does work on a particle only when the particle undergoes a displacement in the direction of the force
- Consider a force **F** that causes a particle to move along a path *s* from position **r** to new position **r**'.
- If the magnitude of $d\mathbf{r}$ is ds. The angle between $d\mathbf{r}$ and **F** is θ.
- By definition the work done by **F** is

$$dU = F\, ds \cos\theta$$

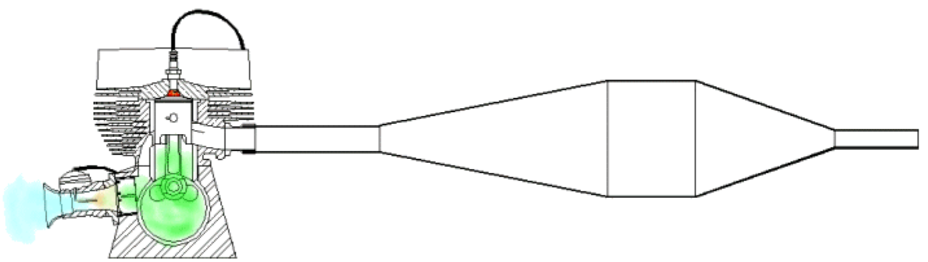


The Work of a Force

- We may also use the (vector) dot product

$$dU = \mathbf{F}.\, d\mathbf{r}$$

- The unit of work is joule (J) which is the work done a 1 Newton force to move a particle through a distance of 1 meter in the direction of the force.

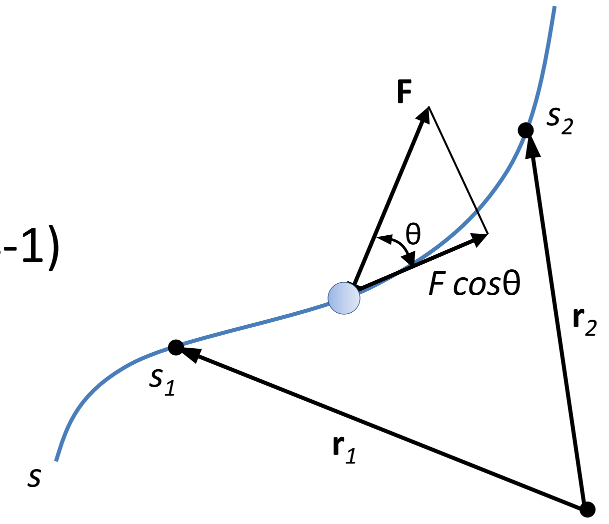


- So 1J = 1 N.m
- In FPS system, work is measured in foot-pound (ft.lb)

Work of a Variable Force

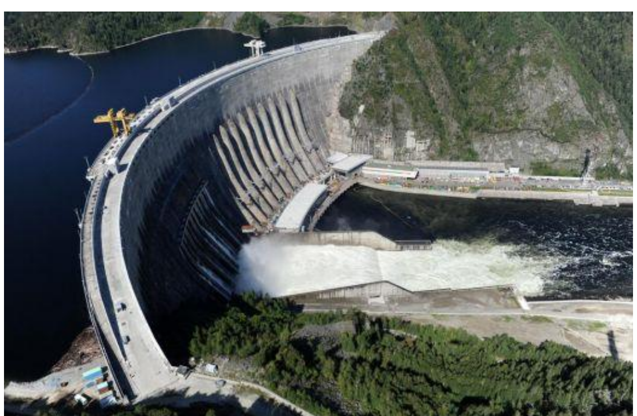
- Consider a particle acted on by a force **F** undergoes a finite displacement along its path from $\mathbf{r}_1$ to $\mathbf{r}_2$
- The work done can be determined through integration

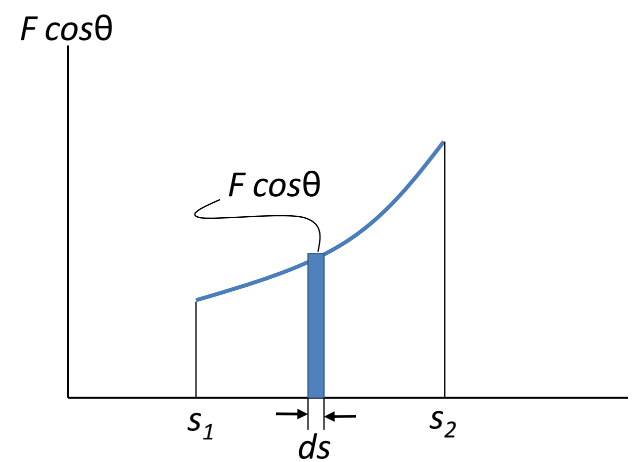
$$U_{1-2} = \int_{r_1}^{r_2} \mathbf{F} . d\mathbf{r} = \int_{s_1}^{s_2} F \cos\theta \, ds \quad (14\text{-}1)$$



Work Done by Variable Force

- Alternately the work can be obtained from a plot of the force component versus position
- The area under the graph bounded by s_1 and s_2 represents the total work done by the force on the particle

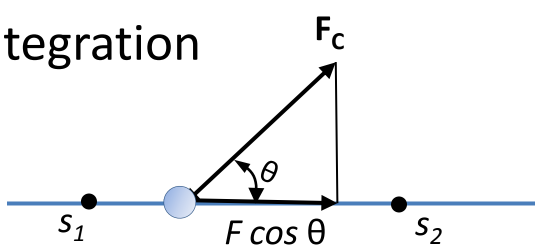




Work of a Constant Force

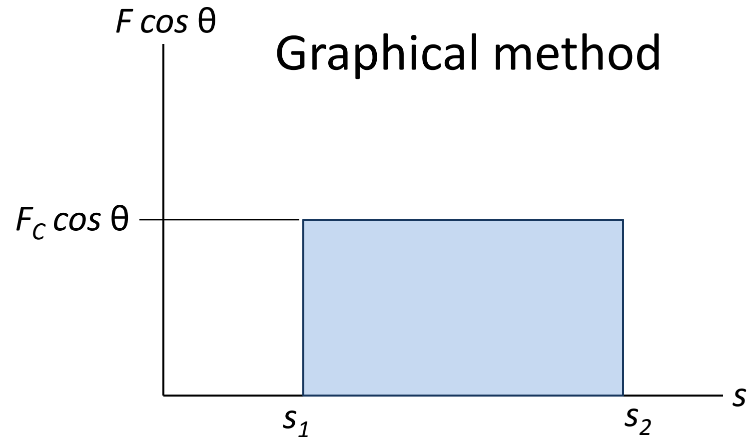
- Consider a constant force moving along a straight line under a constant force $\mathbf{F}_c$ at constant θ to the horizontal

By Integration



$$U_{1-2} = F_C \cos\theta \int_{s_1}^{s_2} ds$$

Graphical method



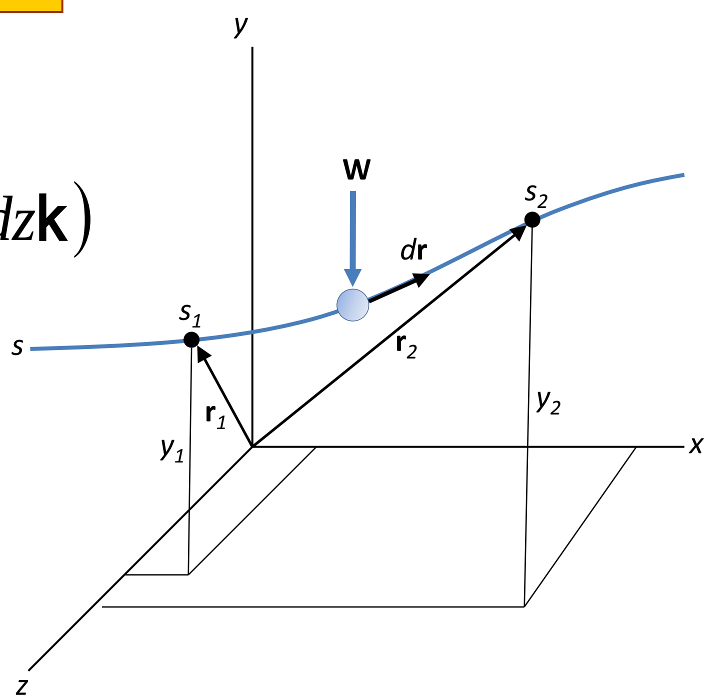
$$U_{1-2} = F_C \cos\theta \left(s_2 - s_1\right) \qquad (14\text{-}2)$$

Work of a Weight

- Consider a particle of weight W moving along a path, from position s_1 to position s_2
- At any point along the path:
- Displacement $\boxed{d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}}$
- Vertical forces $\boxed{\mathbf{W} = -W\mathbf{j}}$
- Work: $U_{1-2} = \int_{r_1}^{r_2} \mathbf{F}.d\mathbf{r} = \int_{r_1}^{r_2} -W\mathbf{j}.(dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k})$

$$U_{1-2} = \int_{y_1}^{y_2} -W\ dy = -W(y_2 - y_1)$$

- or $\boxed{U_{1-2} = -W\,\Delta y}$ (14-3)



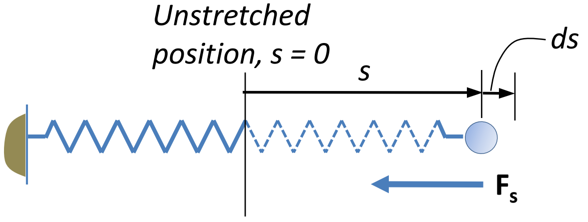
Work of a Spring Force

- If an elastic spring is stretched a distance *ds*, on release, the work done by the force that acts on a particle attached to the spring

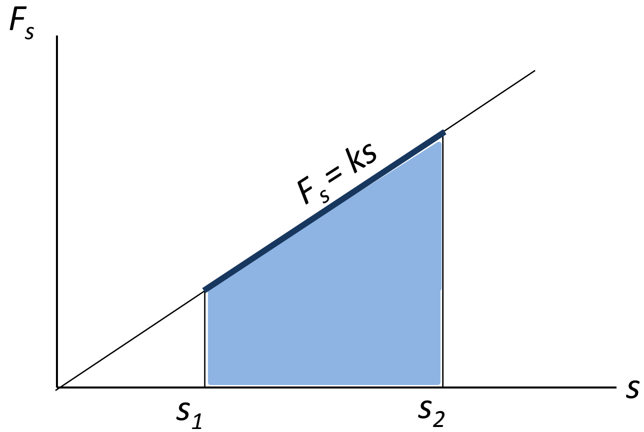
$$U_{1-2} = \int_{s_1}^{s_2} F_s\, ds = \int_{s_1}^{s_2} -ks\, ds$$

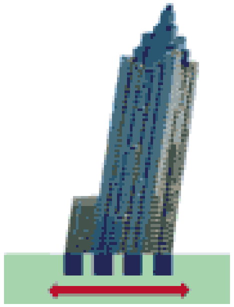
$$U_{1-2} = -\left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right) \qquad (14\text{-}4)$$

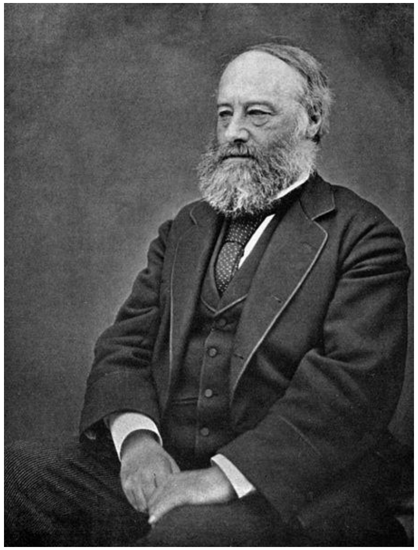
- k is called the spring constant
- Note that for a spring F_s acts in the opposite sense of the displacement *ds*



Work of a Spring Force

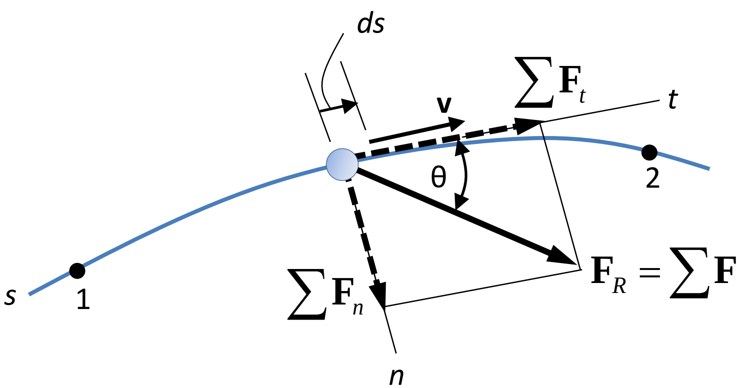






James Joule

Principle of Work and Energy



- Consider a particle of mass *m* moving along path *s* subject a system of external forces with resultant $\mathbf{F}_R$ causing motion from s_1 to s_2 with respective velocities v_1 and v_2

- Applying the Equation of motion in the tangential direction

$$\sum \mathbf{F}_t = m a_t$$

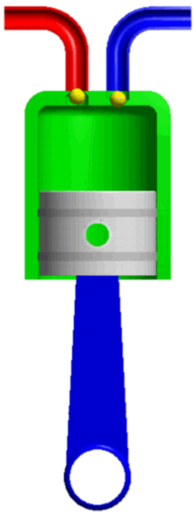
- From the kinematic equation $a_t = v\dfrac{dv}{ds}$

Principle of Work and Energy

$$\sum \mathbf{F}_t = mv\frac{dv}{ds}$$

$$\sum \int_{s_1}^{s_2} \mathbf{F}_t \, ds = \int_{v_1}^{v_2} mv \, dv$$

$$\sum \int_{s_1}^{s_2} \mathbf{F}_t \, ds = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$$



- So work done between s_1 and s_2 is;

$$\boxed{\sum U_{1-2} = \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2} \qquad (14\text{-}6)$$

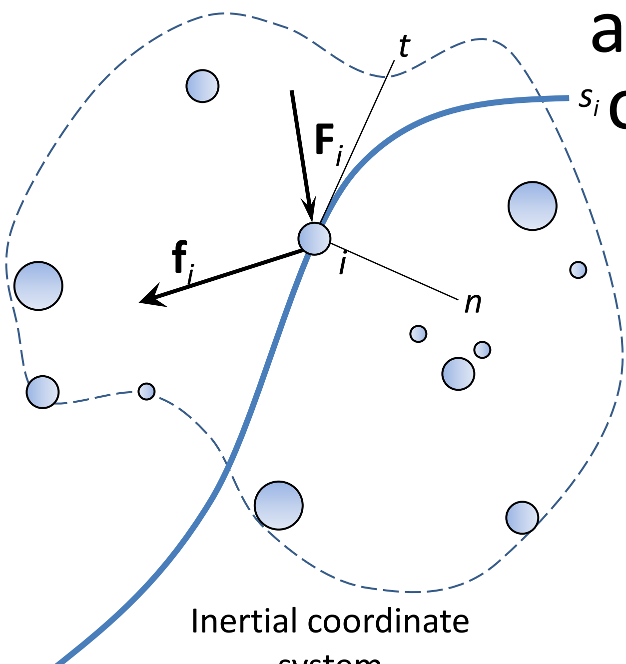
Principle of Work and Energy

- Eqn 14-6 represents the *principle of work and energy*
- Let $T = \frac{1}{2}mv^2$ *T* is called the *kinetic energy* of the particle
- The work done on a particle causes a change in the energy of the particle
- Energy is a scalar with units of Joules (J)
- Eqn 14-6 can re-written as

$$\boxed{T_1 + \sum U_{1-2} = T_2} \qquad (14\text{-}7)$$

Work & Energy for a System of Particles

- The principle of work and energy can be extended to a system of particles (Sec 13.3) confined in a region of space
- If $\mathbf{F}_i$ and $\mathbf{f}_i$ are the resultants of the external and internal forces respectively on particle i , then



$$\boxed{\sum T_1 + \sum U_{1-2} = \sum T_2} \qquad (14\text{-}8)$$

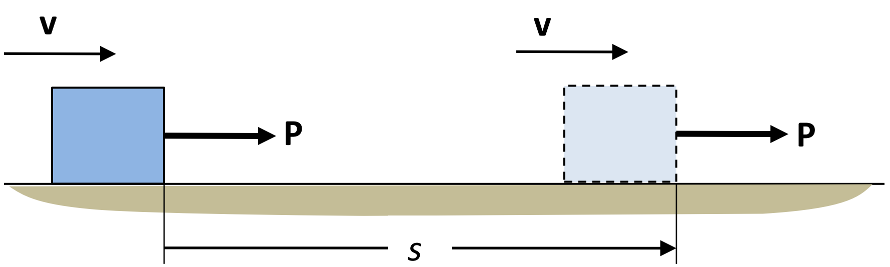
- In other words, the final kinetic energy of the system is the initial plus the all work done on all particles by all relevant forces

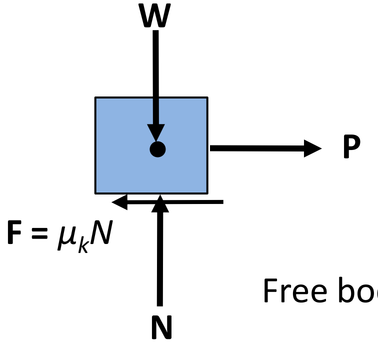
Work & Energy for a System of Particles

- Notes:
  - If the system is a *translating rigid body* then all particles will undergo the same displacement
  - If the system is a *nonrigid body*, then the constituent particles of the body may undergo their own displacements along different paths

Work Friction Caused by Sliding

- Consider a block sliding over a distance *s* on a rough surface due to an applied force P which is just enough to counteract the frictional resistance F and maintain constant velocity v





Free body diagram

Work Friction Caused by Sliding

- Applying Eqn 14-8:

$$\tfrac{1}{2}mv^2 + \left(Ps - \mu_k Ns\right) = \tfrac{1}{2}mv^2$$

- The final kinetic energy of our system is the original plus the work done by applied force plus work done by friction (negative due to its resistive action)
- Note that the friction component includes external work done by friction and the internal work which is converted into some form of internal energy such as heat

Power & Efficiency

- How much work can be done or delivered in a given amount of time?
- The rate of work is called *power*
- The *power* generated by a machine that performs work *dU* in a time interval *dt* is therefore

$$P = \frac{dU}{dt} \qquad (14\text{-}9)$$

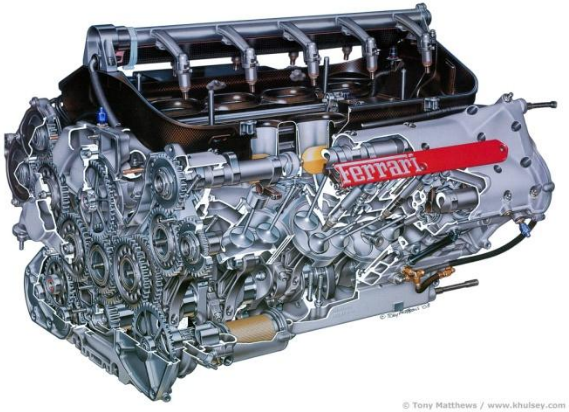
Power

- If a force **F** causes a displacement *d***r** over time interval *dt*, then

$$P = \frac{dU}{dt} \qquad = \frac{\mathbf{F} \cdot d\mathbf{r}}{dt}$$

$$\boxed{P = \mathbf{F} \cdot \mathbf{v}} \qquad (14\text{-}10)$$

- The unit of power is the Watt (W) in SI or horsepower (hp) in FPS system
- 1W = 1 J/s = 1 N .m/s
- I hp = 550 ft. lb/s;   1 hp = 746 W



Efficiency

- *Efficiency* is defined as the ratio of the output of useful power produced by a machine to the input of power supplied to the machine

$$\varepsilon = \frac{power \quad output}{power \quad input} \qquad (14\text{-}11)$$

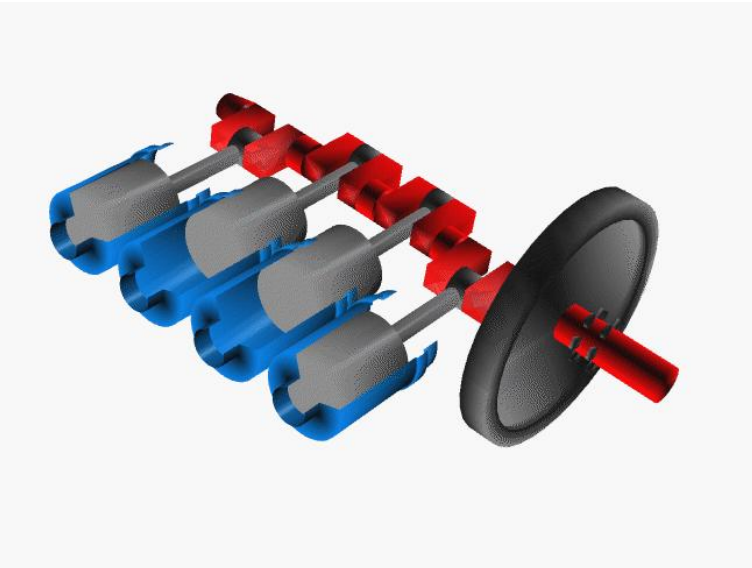
- If the energy supplied and the energy output occur within the *same time interval*, then

$$\varepsilon = \frac{energy \quad output}{energy \quad input} \qquad (14\text{-}12)$$

Efficiency

- A *mechanical efficiency* of 1 would indicate perfection. Due to frictional forces and other inherent factors that result in some power losses, this is not achievable
- Efficiency of a machine will always be less than 1.





Conservative Forces & Potential Energy

- If the work of a force is dependant only on the initial and final positions and not the path, then the force is a *conservative force*.
- Examples
  - Gravitational force on a particle, aka weight
  - Work done by a spring that has been elongated or compressed
- *Energy* is defined as the capacity to do work

Energy

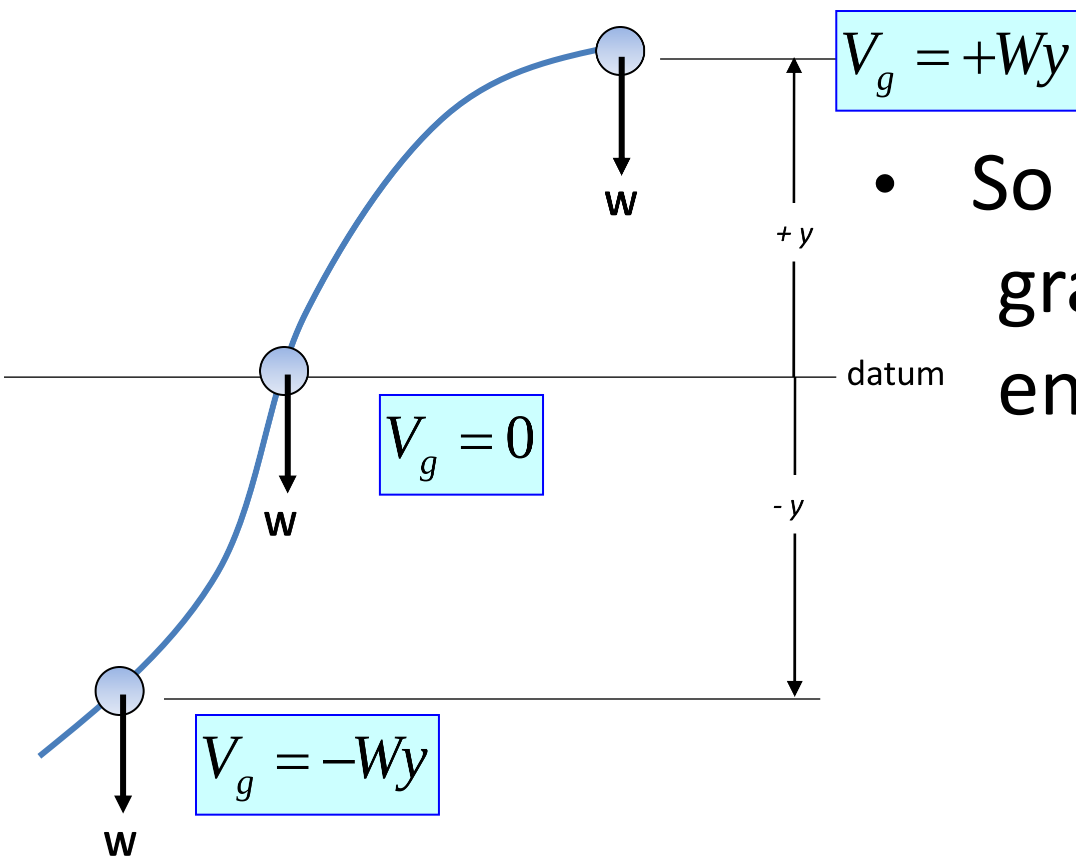
- Consider a particle at rest, it needs some energy to be imparted to enable it move at a given speed (kinetic energy).
- Once in motion, now this particle now performs work and can do work on some other particle.
- The kinetic energy it gained gave it a capability to do work elsewhere. This capability is called *Energy*

Potential Energy

- When a particle has energy due to its position above a datum, it is called *potential energy*
- When dealing with the weight of a particle above a datum we refer to its *gravitational potential energy*
- If the particle is attached to an elastic spring then we are dealing with *elastic potential energy*

Gravitational Potential Energy

- The gravitational potential energy is the work to be done to get to the datum

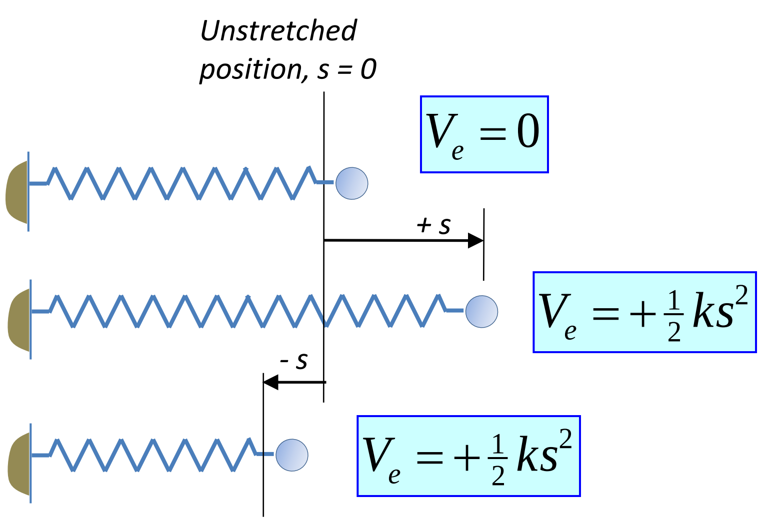


- So generally, the gravitational potential energy

$$V_g = Wy \qquad (14\text{-}13)$$

Elastic Potential Energy

- When a elastic spring *k*, is elongated or compressed a distance *s*, elastic potential energy V_e is imparted to the spring.
- By definition the elastic potential energy is always a positive value



$$\boxed{V_e = +\tfrac{1}{2}ks^2} \qquad (14\text{-}14)$$

Potential Function

- If a particle is subjected to both gravitational and elastic forces, the potential energy components can be summed into a *potential function*.

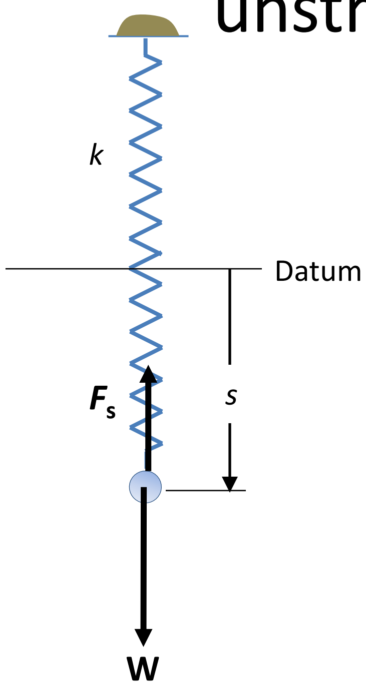
$$\boxed{V = V_g + V_e} \qquad (14\text{-}15)$$

- The work done a by a conservative force moving a particle from a point 1 to a point 2 is the *difference* of the potential function at the two points

$$\boxed{U_{1-2} = V_1 - V_2} \qquad (14\text{-}16)$$

Potential Function

- Consider a particle suspended on a spring and at a distance *s* from a datum located at the unstretched length of the spring



$$V = V_g + V_e$$

$$V = -Ws + \tfrac{1}{2}ks^2$$

- If the particle were to move from a position s_1 to a position s_2, then the work done

$$U_{1-2} = V_1 - V_2$$

$$U_{1-2} = (-Ws_1 + \tfrac{1}{2}ks_1^{\,2}) - (-Ws_2 + \tfrac{1}{2}ks_2^{\,2})$$

Potential Function

$$U_{1-2} = W(s_2 - s_1) - (\tfrac{1}{2}ks_2{}^2 - \tfrac{1}{2}ks_2{}^2)$$

- For an infinitesimal displacement from a point (*x, y, z*) to (*x + dx, y + dy, z + dz*), Eqn 14-16 becomes

$$dU = V(x, y, z) - V(x + dx, y + dy, z + dz)$$

$$dU = -dV(x, y, z) \qquad \text{(14-17)}$$

- If we represent force and distance as vectors

$$dU = \mathbf{F}.d\mathbf{r} = (F_x\mathbf{i} + F_y\mathbf{j} + F_z\mathbf{k}).(dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k})$$

$$dU = \mathbf{F}.d\mathbf{r} = (F_x dx + F_y dy + F_z dz)$$

Potential Function

- Comparing with Eqn 14-17,

$$F_x dx + F_y dy + F_z dz = -dV(x, y, z)$$

- Applying partial derivatives (students please review)

$$F_x dx + F_y dy + F_z dz = -\left(\frac{\partial V}{\partial x} dx + \frac{\partial V}{\partial y} dy + \frac{\partial V}{\partial z} dz \right)$$

$$F_x = -\frac{\partial V}{\partial x}, \quad F_y = -\frac{\partial V}{\partial y}, \quad F_z = -\frac{\partial V}{\partial z} \qquad \text{(14-18)}$$

Potential Function

- So that

$$\mathbf{F} = -\left(\frac{\partial V}{\partial x}\mathbf{i} + \frac{\partial V}{\partial y}\mathbf{j} + \frac{\partial V}{\partial z}\mathbf{k} \right)$$

$$\mathbf{F} = -\left(\frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k} \right)V$$

$$\mathbf{F} = -\nabla V \qquad \text{(14-19)}$$

- Eqn 14-18 (or 14-19) provide the criterion to test whether **F** is indeed a conservative force or not.

Principle of Conservation of Energy

- Let us expand Eqn 14-16 for a particle subjected to both conservative and nonconservative forces

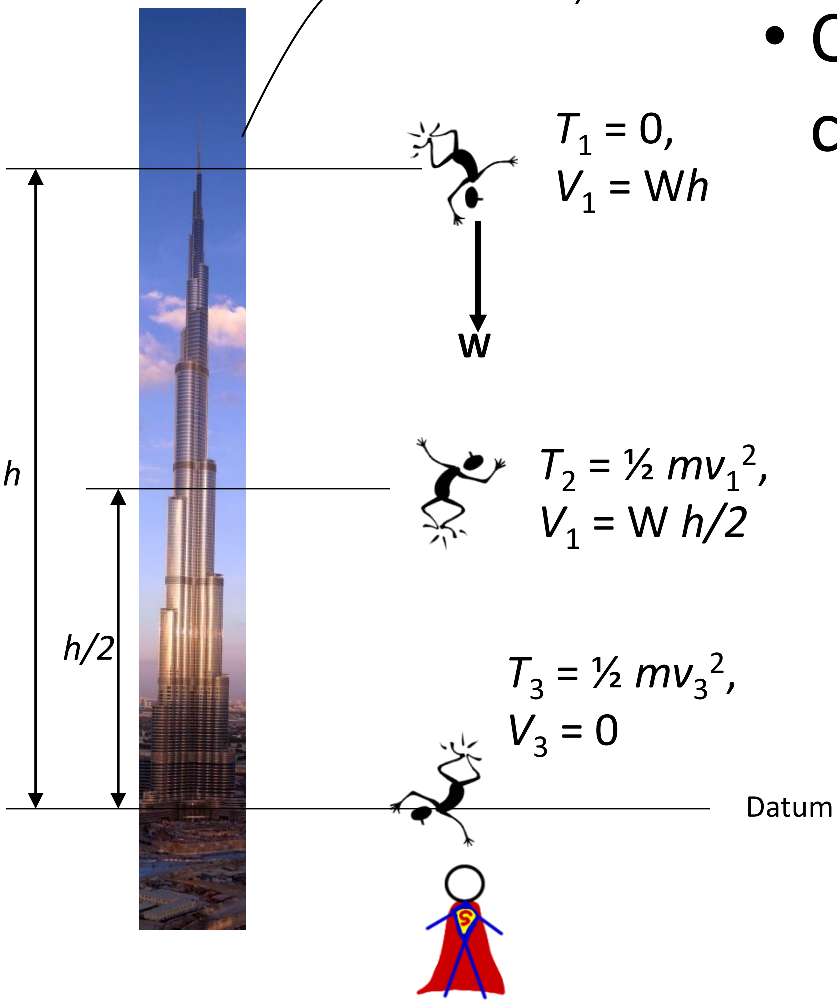
$$T_1 + V_1 + \left(\sum U_{1-2}\right)_{nonconservative} = T_2 + V_2 \quad (14\text{-}20)$$

- In other words during the motion, the sum total of all the particle's energies remains constant.
  Eqn 14-20 is the *Principle of conservation of mechanical energy*.
- In other words a particle's energy does not simply "go away", but rather it is transformed from one from one type to another

Principle of Conservation of Energy

- Consider a ball dropped from a height *h* above a datum

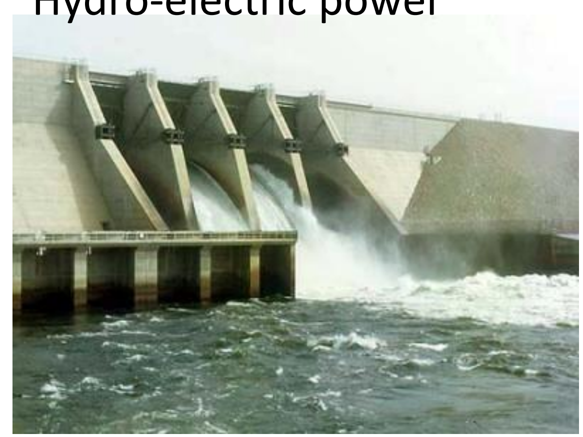
The Burj Khalifa, world's tallest building,
Dubai, United Arab Emirates



- Considering only the conservative forces, from 14-20

$$\boxed{T_1 + V_1 = T_2 + V_2} \quad (14\text{-}21)$$

Real life example:
Hydro-electric power



Principle of Conservation of Energy

- We can apply Eqn 14-21 to any two locations of the flight path we may want to analyze.
- For 1 and 2:

$$0 + Wh = \tfrac{1}{2} m v_2^2 + W \frac{h}{2}$$

- For 1 and 3: Just before impact, all the original potential energy has been converted to kinetic energy

$$0 + Wh = \tfrac{1}{2} m v_3^2 + 0$$

- So what *v* of the “skydiver” would Superman be dealing with? Confirm your answer using 2 and 3.

Principle of Conservation of Energy For a System of Particles

- If we have more than one particle in our system of interest then we can expand Eqn 14-20 to accommodate them, as follows

$$\boxed{\sum T_1 + \sum V_1 + \left(\sum U_{1-2}\right)_{nonconservative} = \sum T_2 + \sum V_2}$$

- Or if the nonconservative forces are not relevant

$$\boxed{\sum T_1 + \sum V_1 = \sum T_2 + \sum V_2} \quad (14\text{-}22)$$

Questions