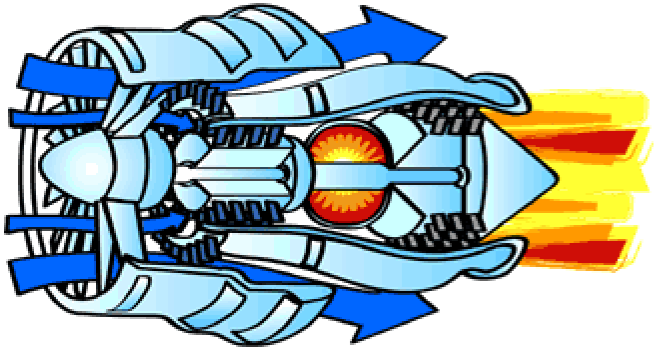


Kinetics of a Particle

Impulse & Momentum I

Outline

- Principle of Linear Impulse and Momentum
- Mechanics of Impact

Principle of Linear Impulse and Momentum

- From Newton's 2nd law

$$\boxed{\sum \mathbf{F} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt}} \qquad (15\text{-}1)$$

$$\sum \mathbf{F}dt = md\mathbf{v}$$

- If v_1 and v_2 are the velocities at times t_1 and t_2 respectively, then

$$\sum \int_{t_1}^{t_2} \mathbf{F}dt = \int_{v_1}^{v_2} md\mathbf{v}$$

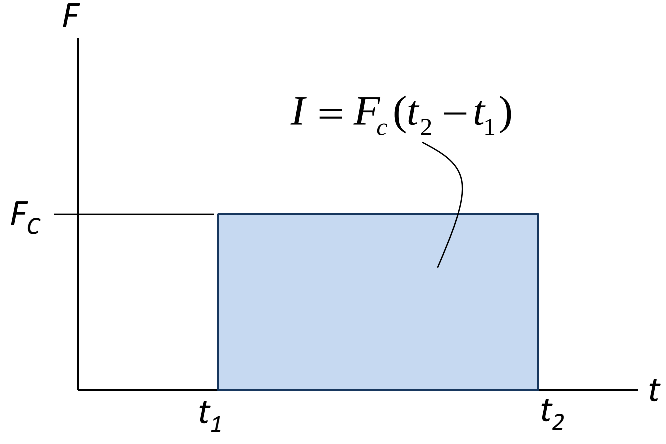
$$\boxed{\sum \int_{t_1}^{t_2} \mathbf{F}dt = mv_2 - mv_1} \qquad (15\text{-}2)$$

Principle of Linear Impulse and Momentum

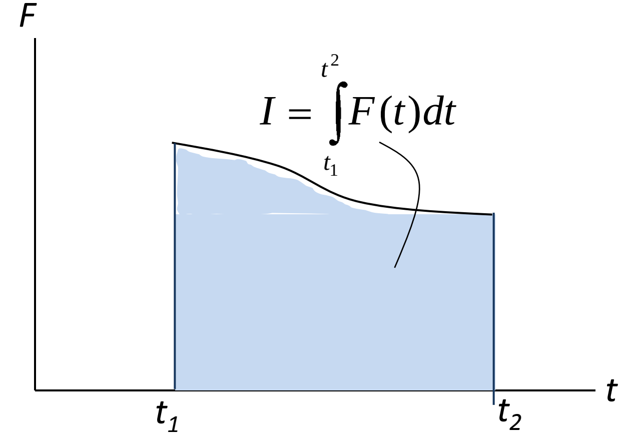
- Eqn 15-2 is referred to as the Principle of Linear Impulse and Momentum
- The LHS is the impulse (**I**), or the sum total effect of the force over time interval $t_2 - t_1$
- Units: based on force-time. N.s or lb.s
- The RHS is the change in momentum (**L** = *mv*) from position 1 to position 2 over the same time interval
- Units: based on mass-velocity. kg.m/s or slug.ft/s

Principle of Linear Impulse and Momentum

- This principle can represented graphically as follows:



Constant force



Variable force

Principle of Linear Impulse and Momentum

- The Principle Linear Impulse and Momentum may be expressed as

$$\boxed{mv_1 + \sum \int_{t_1}^{t_2} \mathbf{F}\,dt = mv_2} \qquad (15\text{-}3)$$

- In other words initial momentum at time t_1 plus the sum total of all applied impulses over time interval $t_2 - t_1$ equals final momentum at time t_2

Principle of Linear Impulse and Momentum

- For the three dimensional components of the impulse and momentum

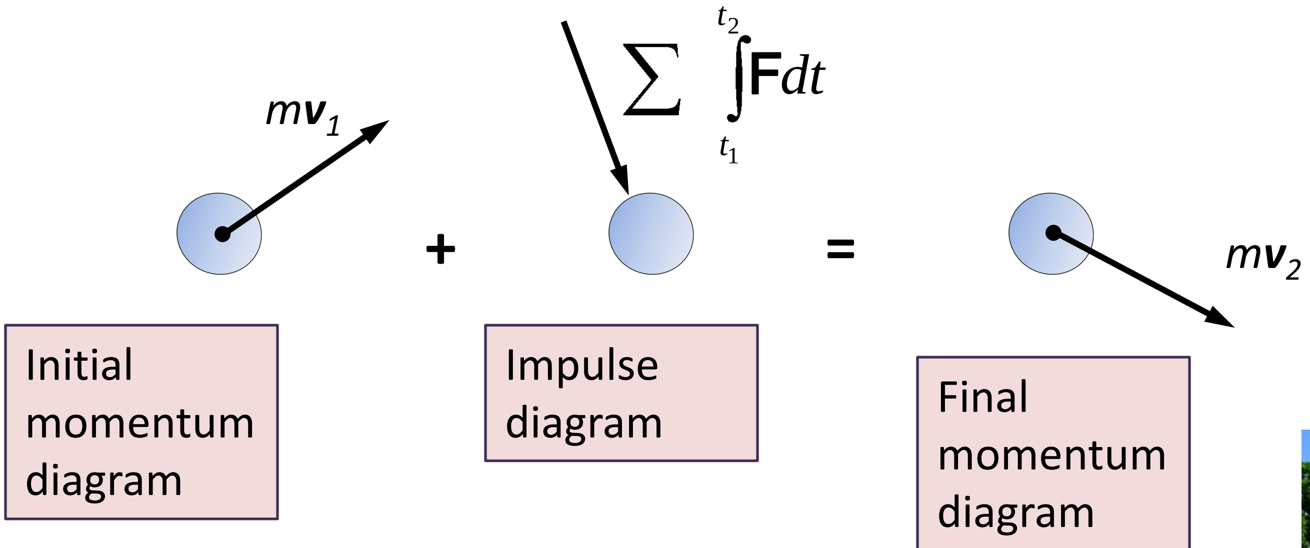
$$m(v_x)_1 + \sum \int_{t_1}^{t_2} F_x \, dt = m(v_x)_2$$

$$m(v_y)_1 + \sum \int_{t_1}^{t_2} F_y \, dt = m(v_y)_2 \qquad (15\text{-}4)$$

$$m(v_z)_1 + \sum \int_{t_1}^{t_2} F_z \, dt = m(v_z)_2$$

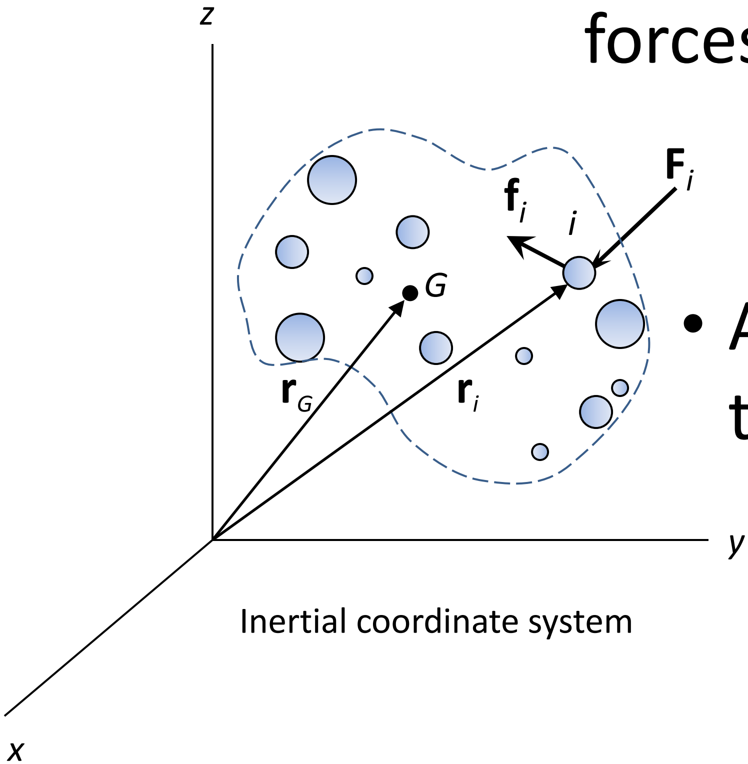
Principle of Linear Impulse and Momentum

- Eqn 15-3 may be represented graphically a follows



Principle of Linear Impulse and Momentum for a System of Particles

- Consider a system of particles confined within a region of space. If $\mathbf{F}_i$ is the resultant of the internal forces acting on particle *i*, and $\mathbf{f}_i$ the resultant of the internal forces acting on particle *i*



Inertial coordinate system

- Applying the Equation of motion to the individual particles

$$\sum \mathbf{F}_i = \sum m_i \frac{dv_i}{dt} \qquad (15\text{-}5)$$

Principle of Linear Impulse and Momentum for a System of Particles

- The internal forces sum up to cancel themselves out [Students prove on your own]
- Integrating Eqn 15-5

$$\boxed{\sum m(v_i)_1 + \sum \int_{t_1}^{t_2} F_i\,dt = \sum m(v_i)_2} \quad \text{(15-6)}$$

- We may also determine the center of mass of the system of particles from

$$m\mathbf{r}_G = \sum m_i \mathbf{r}_i$$

- By differentiating the position vector, we obtain the velocity

$$m\mathbf{v}_G = \sum m_i \mathbf{v}_i$$

Principle of Linear Impulse and Momentum for a System of Particles

- Since the center of mass represents all the particles in the system, we may re-write Eqn 15-6 as

$$\sum m(v_G)_1 + \sum \int_{t_1}^{t_2} F_i\,dt = \sum m(v_G)_2 \qquad (15\text{-}7)$$

- So the initial aggregate momentum plus the sum total of all external impulses equals the final aggregate momentum
- Therefore the above equations are applicable if we had analyzed the system of particles as a rigid body composed of *i* particles

Conservation of Linear Momentum for a System of Particles

- Assuming there are no external impulses applied on our system of particles, Eqn 15-6 would become

$$\boxed{\sum m(\mathbf{v}_i)_1 = \sum m(\mathbf{v}_i)_2} \qquad (15\text{-}8)$$

- In other words the total linear momentum remains constant
- Using the center of mass

$$m\mathbf{v}_G = \sum m_i \mathbf{v}_i$$

$$\boxed{(\mathbf{v}_G)_1 = (\mathbf{v}_G)_2} \qquad (15\text{-}9)$$

Conservation of Linear Momentum for a System of Particles

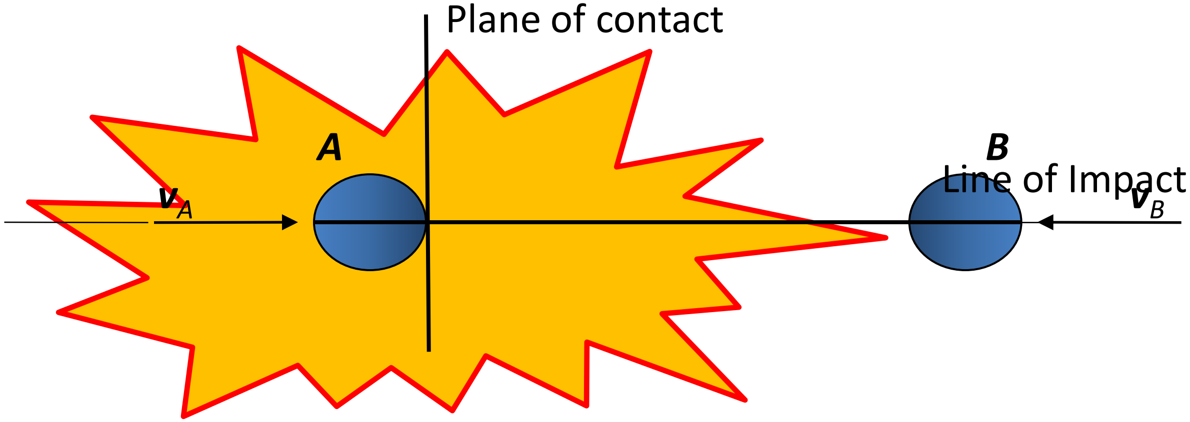
- Eqn 15-9 indicates the motion of the center of mass for the system of particles does not change if no external impulses are applied to the system

Impact

- *Impact* occurs when two bodies collide during a very short period of time , causing relatively large forces to be exerted between the bodies
- Examples;
  - Striking a nail with a hammer
  - Golf club on a golf ball
  - Traffic crash
  - Pile driver

Impact

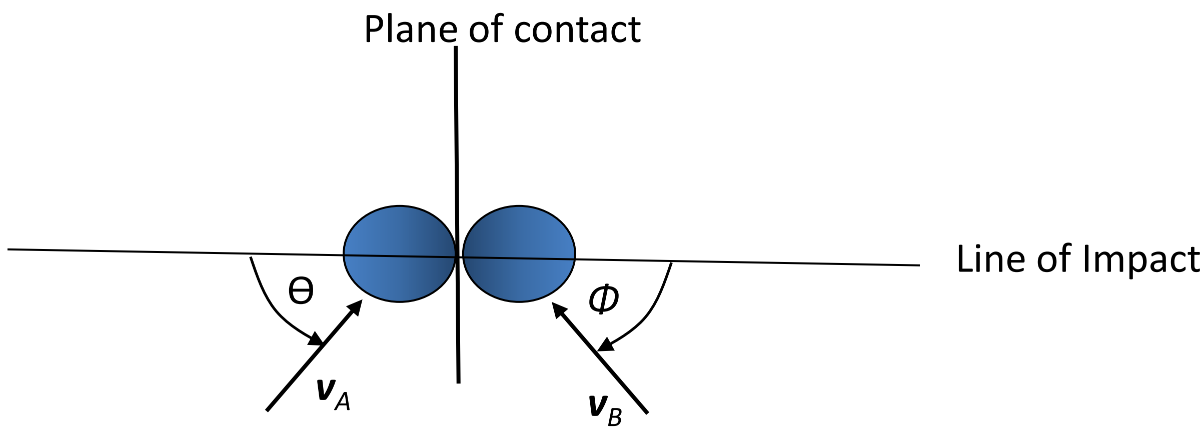
- Generally two types of impact
- *Central impact:* when the direction of motion of the centers of mass of the two colliding particles is along the line though the centers of mass of the particles, also known as the *line of impact*



- Line of impact is always perpendicular to the *plane of contact*

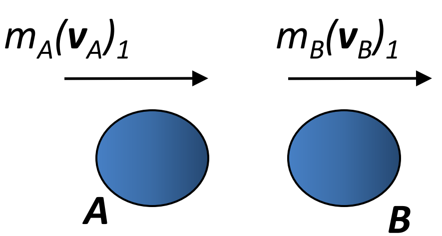
Oblique Impact

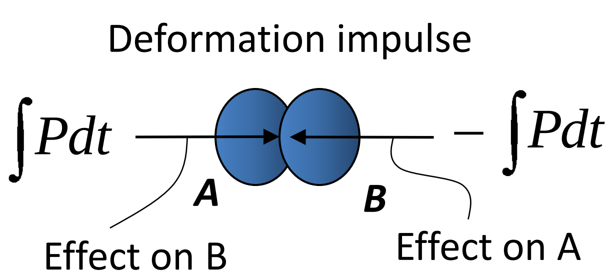
- If one of the particles' motion makes an angle with the line of impact then we have an *oblique impact*

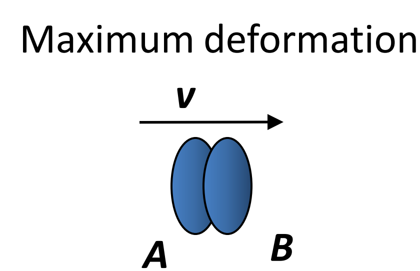


Mechanics of Central Impact

- Consider two particles with initial momenta as indicated
- If $(v_A)_1 > (v_B)_1$, then at some point impact will occur
- Assuming the particles are *deformable*, they will undergo a *period of deformation* during which they exert equal but opposite *deformation impulse* on each other.
- At instant of maximum deformation, relative motion is zero, hence they move with a common velocity

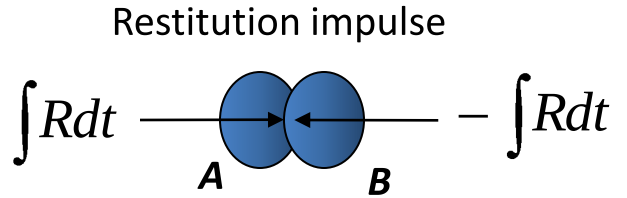






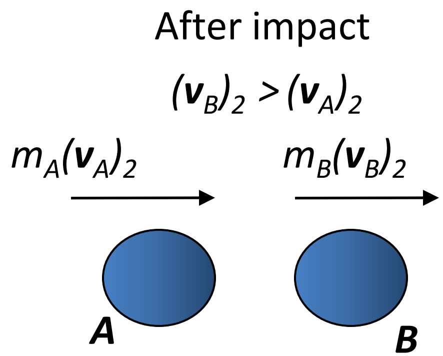
Mechanics of Central Impact

- A period of *restitution* then occurs during which the particles return to original shape or some degree of it.
- If the latter happens then the particles will have some *permanent deformation*
- It is the *restitution impulse* that pushes the particles apart and attempts to regain the initial shape
- In reality deformation impulse is always greater than the restitution impulse. Why?



Mechanics of Central Impact

- After separation, the particles will have their final momenta, where $(v_B)_2 > (v_A)_2$



- Problem solving: The initial velocities are known and the objective will be to determine the final velocities
- Problem solving: The *momentum of the system is conserved*, and the effects of internal deformation and restitution can be neglected.

Analysis of Impact

- For the two particle undergoing central impact
- By the Principle of Conservation of Impulse and Momentum

$$m_A (v_A)_1 + m_B (v_B)_1 = m_A (v_A)_2 + m_B (v_B)_2 \quad (15\text{-}10)$$

- Applying the principle of conservation of Impulse and Momentum to the deformation of particle A

$$m_A (v_A)_1 - \int P dt = m_A v$$

Analysis of Impact

- Applying the principle of conservation of Impulse and Momentum to the restitution of particle A

$$m_A v - \int R\,dt = m_A (v_A)_2$$

- The ratio of the restitution to impulse is called the *coefficient of restitution*, *e*. So for particle A;

$$e = \frac{\int R\,dt}{\int P\,dt} = \frac{v - (v_A)_2}{(v_A)_1 - v}$$

Analysis of Impact

- Likewise for particle B

$$e = \frac{\int R\,dt}{\int P\,dt} = \frac{(v_B)_2 - v}{v - (v_B)_1}$$

- Going through some more steps it can be shown that

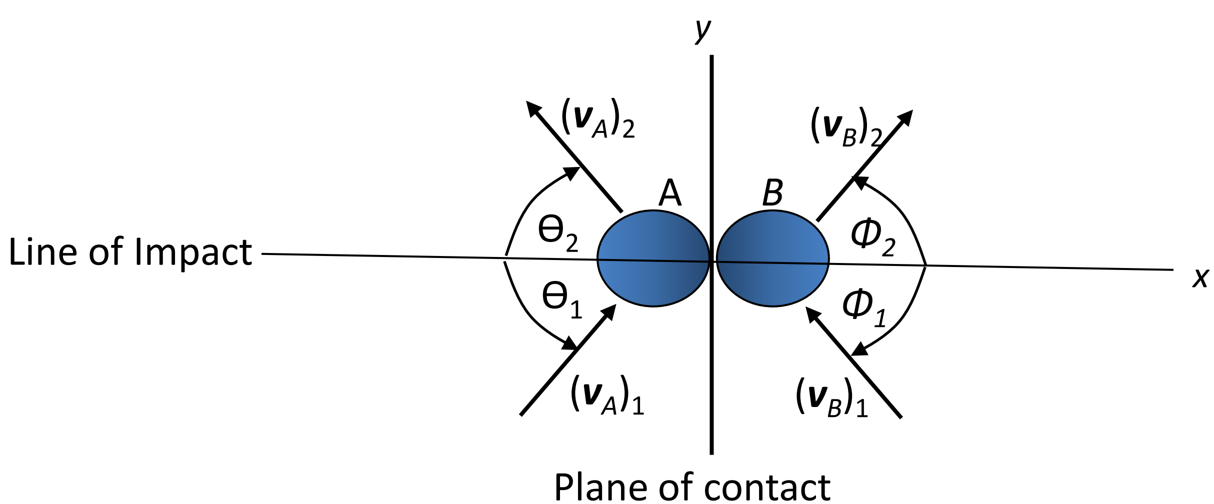
$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} \qquad (15\text{-}11)$$

Coefficient of Restitution

- *e* = 1 implies *perfectly elastic* collision. In reality this has never been achieved, due to?
- *e* = 0 implies *inelastic* or *plastic* collision. The two particles stick together and move with a common velocity after impact
- Problem solving! If *e* is not given, it is often necessary to use the conservation of kinetic energy of the system of particles to help us solve impact problems [review Eqn 14-8]
- *e* = 1 implies no energy loss on impact
- *e* = 0 implies maximum energy loss on impact

Oblique Impact

- For oblique impact the procedures of central impact are conducted separately for the *x* and *y* components
- The resultant velocity after impact for each particle enables us to obtain the direction and magnitude of the velocities (and momenta) after impact.



Questions & Comments

- Examples