Rotation About Fixed Axis

16 – 2

$$\alpha = 20e^{-0.6t}$$

$$\alpha = \frac{d\omega}{dt}$$, where ω is angular velocity

so

$$\frac{d\omega}{dt} = 20e^{-0.6t}$$

$$d\omega = 20e^{-0.6t}\,dt$$

$$\int_0^{\omega} d\omega = \int_0^3 20e^{-0.6t}\,dt$$

$$\omega = 20\left[\frac{1}{-0.6}e^{-0.6t}\right]_0^3$$

$$\omega = -\frac{20}{0.6}\left[e^{-0.6(3)} - e^{-0.6(0)}\right]$$

$$= -33.33\left[0.1652 - 1\right]$$

$$= +27.82 \text{ rad/s}$$

Speed $V_p = \omega r = 27.82(1.75) = 48.65$ ft/s.

one revolution is 2π rad.

$$\omega = \frac{d\theta}{dt}$$

$$d\theta = \omega \, dt$$

$$\int d\theta = \int \omega \, dt$$

$$\int_0^{\theta} d\theta = \int_0^{t=3} \omega \, dt$$

$$\theta = \int_0^{t=3} -\frac{20}{0.6} e^{-0.6t}$$

$$= -33.33 \left[\frac{1}{-0.6} e^{-0.6t} \right]_0^3$$

$$= +55.55 \left[e^{-0.6(3)} - e^{0} \right]$$

$$= 55.55 \left[0.1652 - 1 \right]$$

$$= -46.37 \text{ rad}$$

Rotation About Fixed Axis

16 – 10

$$\alpha = 0.2\theta$$

~~$$\alpha = \frac{d\omega}{dt}$$~~

~~$$d\omega = \alpha\, dt$$~~

~~$$\int_{\omega_1}^{\omega_2} d\omega = \int$$~~

$$\alpha\, d\theta = \omega\, d\omega$$

$$0.2\theta\, d\theta = \omega\, d\omega$$

$$0.2\int_{\theta_1}^{\theta_2} \theta\, d\theta = \int_{\omega_1}^{\omega_2} \omega\, d\omega$$

$$0.2\int_{0}^{2(2\pi)} \theta\, d\theta = \int_{5}^{\omega} \omega\, d\omega$$

$$\frac{0.2}{2}\left[\theta^2\right]_0^{4\pi} = \frac{1}{2}\left[\omega^2\right]_5^{\omega}$$

$$0.1\,(4\pi)^2 = 0.5(\omega^2 - 5^2)$$

$$0.1\,(16\pi^2) = 0.5(\omega^2 - 25)$$

$$\omega = ~~8.03~~\ 7.52 \text{ rad/s}$$

Speed at point P $= \omega r = 7.52(2.5)$

~~$= 8.03(2.5)$~~

~~$= 20.58$ ft/s~~

$= 18.8$ ft/s.

16 - 9

$$\alpha_A = 3t + 2$$

$$\alpha_A = \frac{d\omega_A}{dt} = 3t + 2$$

$$\int_{\omega_1}^{\omega_2} d\omega_A = \int_{t_1}^{t_2} (3t+2)\,dt$$

$$\int_{0}^{\omega} d\omega_A = \int_{0}^{5} (3t+2)\,dt$$

$$\omega_A = \left[\frac{3t^2}{2} + 2t\right]_0^5$$

$$= \frac{3(5)^2}{2} + 2(5)$$

~~= 188 rad/s~~

$$= 47.5 \text{ rad/s}$$

This will be transferred to C since C is same size as A

Note that C and B will have same tangential speed

So tangential velocity (Speed)

$$V_C = V_B = \omega_C r_C = \omega_B r_B$$

$$47.5(50) = \omega_B(75)$$

$$\Rightarrow \quad \omega_B = 31.67 \text{ rad/s.}$$

Absolute Motion

16 – 48

rope pulled at 0.5 m/s.

this pull causes end of beam to have displacement of arc length $\frac{r\theta}{t}$

so a small pull ds, will result in arc displacement $r\,d\theta$

$$ds = r\,d\theta$$

$$\Rightarrow \frac{ds}{dt} = r\frac{d\theta}{dt}$$

$$0.5 = 6\frac{d\theta}{dt} \quad \Rightarrow \frac{d\theta}{dt} = \omega = \frac{1}{12}$$

(constant)

$$\frac{0.5}{6}\,dt = d\theta$$

$$d\theta = ~~\cos~~\frac{1}{12}\,dt$$

$$\int_0^\theta d\theta = \frac{1}{12}\int_0^t dt$$

$$\theta = \frac{1}{12}t$$

Angular acceleration will be zero because we have constant angular velocity

Absolute Motion

16 – 51

$$ds = r\,d\theta$$

$$\frac{ds}{dt} = \frac{r\,d\theta}{dt}$$

$$\frac{ds}{dt} = r\omega$$

$$1 = 12\,\omega$$

$$\omega = \frac{1}{12} \text{ rad/s} \quad \text{or } 0.083 \text{ rad/s}$$

constant ω so angle does not matter

Relative Motion Analysis

F 16-9

We have general plane motion
Select A as the base point
Using vector formulation

$$v_B = v_A + \omega \times r_{B/A}$$

$$4\hat{i} = -2\hat{i} + \omega\hat{k} \times 3\hat{j}$$

($\omega\hat{k}$) acting into the page according to right hand rule

$$\omega\hat{k} \times 3\hat{j} = \begin{vmatrix} i & j & k \\ 0 & 0 & \omega \\ 0 & 3 & 0 \end{vmatrix}$$

$$= (0 - 3\omega)\hat{i} + (0-0)\hat{j} + (0-0)\hat{k}$$

$$= -3\omega\hat{i}$$

$$4\hat{i} = -2\hat{i} - 3\omega\hat{i} = -(2+3\omega)\hat{i}$$

by comparision

$$4 = -(2+3\omega)$$

$$6 = -3\omega$$

$$\omega = -2 \text{ rad/s}$$

so it is actually rotating in the opposite sense (counter clockwise) with my thumb pointed out of page towards me

Relative Motion Analysis

16 − 67

Lets use C and A on the rear wheel with A as base point

$V_C = V_A + \omega \times r_{C/A}$

using vectors

$4\hat{i} = V_A + (-3\omega)\hat{k} \times (26\hat{j})$ — or 2.17 ft

negative because by RHR will be pointing into the page.

$-3\omega\hat{k} \times 26j = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 0 & -3\omega \\ 0 & 2.17 & 0 \end{vmatrix}$

$= 0 + 3\omega \cdot 2.17\,\hat{i} = 6.5\,\hat{i}$

so

so

$4\hat{i} = V_A + 6.5\,\hat{i}$

$V_A = -2.5\,\hat{i}$

← that way