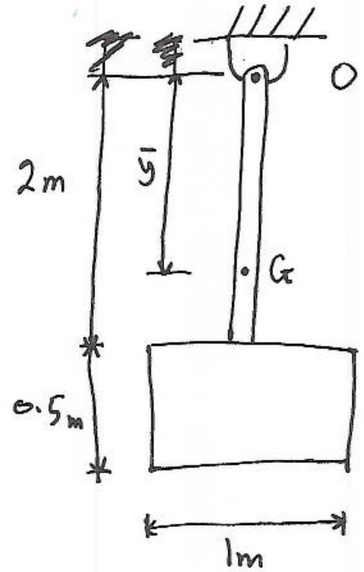
# 17 - 14




for location of center of mass G we need to use the following

$$\sum_{i=1}^{n=2} m_i y_i = m\bar{y}$$

where $m_i$ is the mass of a component called $i$

$y_i$ is the distance of the center of gravity from our reference axis (aka datum), O.

$m$ is the total mass of the system

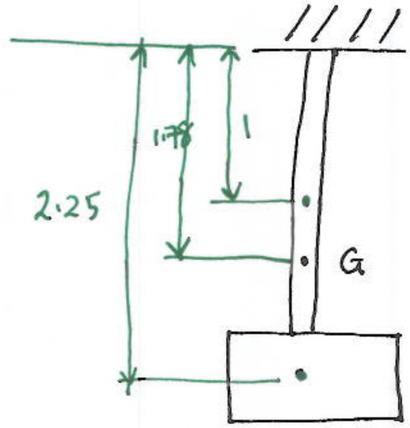
$\bar{y}$ is the distance of the center of mass of the system from the datum O.

$$3(1) + 5(2.25) = (3+5)\bar{y}$$

$$\bar{y} = 1.78 \text{ m}$$

5

We need mass moment of inertia about an axis through G and perpendicular to the page.




So this is a composite, so the MMI is the algebraic sum of the MMI of the components. We can find MMI of each component about its center of mass, then use the Parallel Axis Theorem to convert it to MMI about center of mass of the system G,

Thin plate:

Refer to MMI formulas on inside of back cover of your textbook, otherwise MMI information for common shapes is abundant all over the internet and just about any engineering reference manual. You must match your problem to the correct axis shown on the MMI diagram of that shape. In this problem

$$I_{zz} = \frac{1}{12} m(a^2 + b^2) = \frac{1}{12}(5)(0.5^2 + 1^2) = 0.521 \text{ kg.m}^2$$

We now have to use Parallel Axis Theorem to convert above to rotation about an axis through G.

$$I_{plate, G} = I_g + md^2$$

$$= 0.521 + 5(2.25 - 1.78)^2 = 1.625 \text{ Kg.m}^2$$

Slender rod

$$I_{rod, G} = I_{xx} + md^2$$

$$= \frac{1}{12} ml^2 + m(\bar{y} - y_1)^2$$

$$= \frac{1}{12}(3)(2)^2 + 3(1.78 - 1)^2$$

$$= 2.825 \text{ Kg.m}^2$$

So MMI of this system about axis perpendicular to G

$$I = 0.521 + 2.825 = 3.346 \text{ Kg.m}^2$$

Oops! Hold On! Added wrong number! should be:

$$I = 1.625 + 2.825 = 4.45 \text{ Kg.m}^2$$

# 17 - 46

Speed increased uniformly from rest to 100 m/s over 500 m

$$v^2 = u^2 + 2as$$

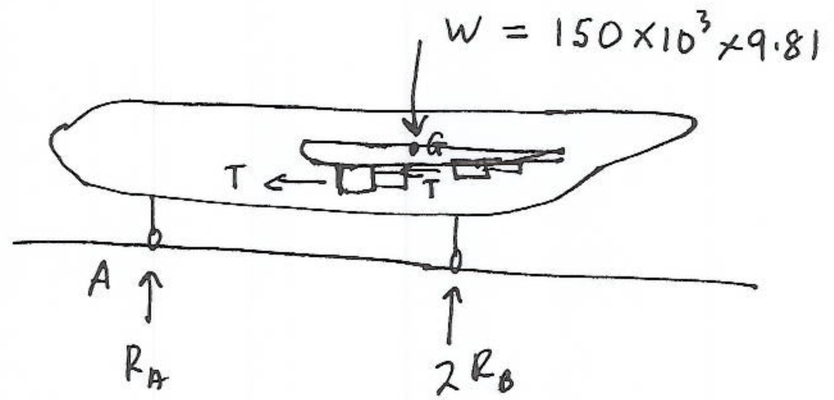
$$a = \frac{v^2 - u^2}{2s} = \frac{100^2 - 0^2}{2(500)} = 10\ m/s^2$$

Applying Equations of Motion - Translation

$$\Sigma F_x = m a_x$$

$$\Sigma F_y = m a_y$$

$$\Sigma M_G = 0$$




because there is another wheel on the other side

Also 2 engines on other wing.

$$\sum F_x = 4T = ma_x$$

$$4T = 150 \times 10^3 \times 10$$

$$T = 375\,000\ N$$

$$= 375\ kN$$

$\sum F_y = 0$ since there is no motion in vertical

$$R_A + 2R_B - W = 0$$

$$R_A + 2R_B = 1471500 \quad \text{— (1)}$$

Let take moments. We can take about G. ~~but that will involve quite a bit. Lets take B.~~ We could also take about A or B or any other convenient point.

$$\sum M_B = 0$$

$$37.5 R_A - 7.5W - 2(4T + 5T) = 0$$

$$R_A = \frac{2(4 \times 375 + 5 \times 375) \times 10^3 + 7.5 \times 150 \times 10^3 \times 9.81}{37.5}$$

$$\sum M_A = 0$$

$$2(4T + 5T) - 30W + 2 \times 37.5 R_B = 0$$

[illegible]

$$R_B = 498600 \text{ N}$$

From Eqn (1)

$$R_A = 1471500 - 2(498600) =$$

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$$\sum M_G = 0$$

$$30 R_A - 7.5 R_B \times 2 + (5T + 4T) \times 2 = 0$$

$$30 R_A - 15 R_B = -6750000 \quad \text{— (2)}$$

30(1) − (2)

$$R_B = 678600$$

$$R_A = 1471500 - 2(678600) = 11430 \text{ N}$$

Note that we could have taken moments about any other point, eg. A, B, ...

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Equations of motion for translation

$$\sum F_x = m a_x$$

$$-2F_B = m a_x$$

$$F_B = -\frac{m a_x}{2} = -\frac{1500\cancel{g} \times 9.81 \times 6}{2\,\cancel{g}} = 44.14\ kN$$

$$\sum F_y = m a_y = 0$$

$$2R_B + R_A = W$$

$$2R_B + R_A = 1500 \times 9.81 = 14{,}715$$

$$\sum M_G = 0$$

$$2 \times 1 \times R_B + 0.25 \times 2 \times F_B \cancel{A} - 2.5 R_A = 0$$

$$\cancel{R_B + 0.5}\ 2R_B - 2.5 R_A = -22070$$

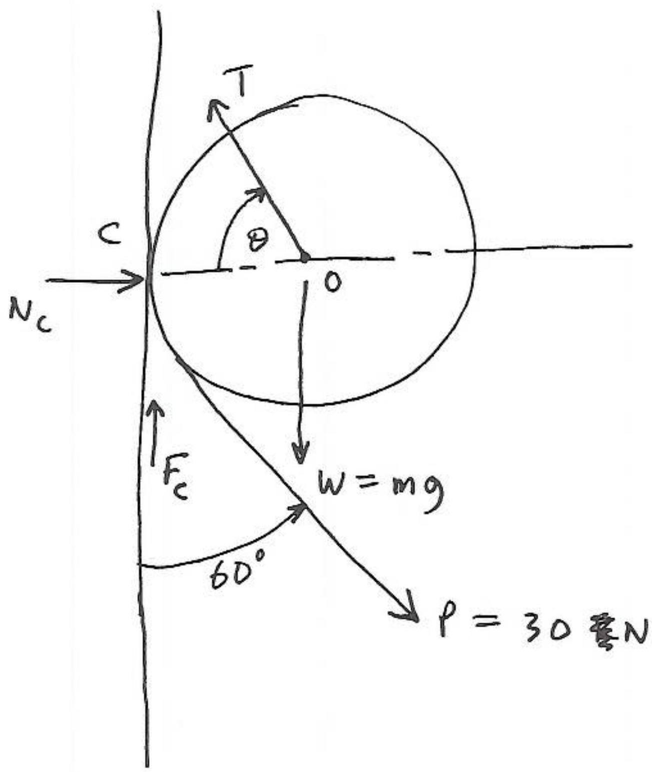
$$R_A = 10510\ N$$

$$R_B = 2102\ N$$

# 17 - 189

Clearly there is no translation. The applied force P causes rotation about O. This causes some tension to be induced in the supporting bracket AB.

So lets draw the free body diagram of the roll.




Since there is no translation

$$\sum F_x = ma_x = 0$$

$$N_c - T\cos\theta + P\sin 60 = 0$$

$$N_c - \frac{5}{13}T + 30\sin 60 = 0$$

$$N_c - 0.384T = -25.98 \quad \text{—— (1)}$$

$$\sum F_y = 0$$

$$T \sin\theta - P\cos 60 + F_c \cancel{\cos\theta} - mg = 0$$

$$\frac{12}{13}T - 30\cos 60 + \mu_c N_c \cancel{\tan\theta} - mg = 0$$

$$0.923T - 30\cos 60 - 17(\cancel{3}9.81) + 0.3N_c = 0$$

$$0.923T + 0.3N_c = 181.77 \quad \text{——(2)}$$

$0.3 \times (1) - (2)$

$$-1.038T = -189.56$$

$$T = 182.62 \text{ N}$$

For angular acceleration $\alpha$ : Take moments about $O$.

$$\sum M_o = I_o \alpha$$

$I_o$ = mass moment of inertia about $O$. (cylinder)

$$= \frac{1}{2} m r^2$$

$$= 0.5(17)(0.12^2) = 0.1224 \text{ kg.m}^2$$

$$\sum M_o = Pr + F_c r$$

$$F_c = 0.3 N_c$$

from Eqn (1)

$$N_c = -25.98 + 0.384T$$

$$= -25.98 + 0.384(182.62)$$

$$= 44.146 \text{ N}$$

so $F_c = 0.3(44.146) = 13.24 \text{ N}$

Therefore

$$\sum M_O = 30(0.12) \cancel{+} - (13.24)(0.12) = \overset{2.0112}{\cancel{6.189}} \text{ Nm}$$

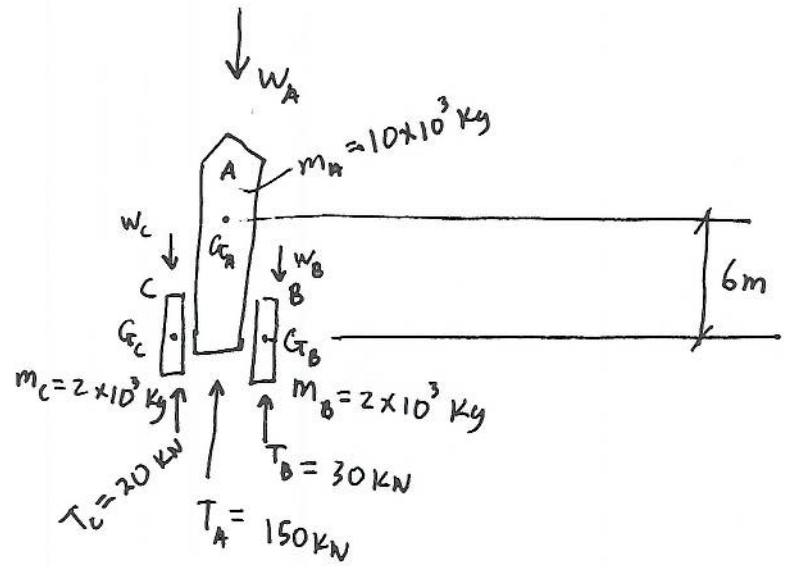
so $\overset{2.0112}{\cancel{6.189}} = 0.1224\,\alpha$

$$\alpha = 16.43 \text{ rad/s}$$

so after all that happened, our roll will end up rotating with angular acceleration $\alpha$ of above.

# 17 - 95

The rocket is rising vertically up (translation). Also due to the differential thrust of the two side boosters the rocket will rotate some (pitch) about an axis pointing out of the page.




Radius of Gyration and Mass moment of inertia. Information given is as follows:

$K_A = 2m$ This is about $G_A$

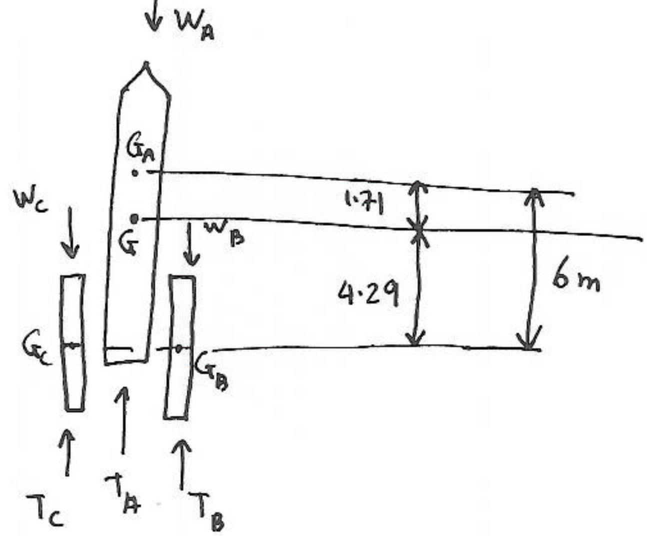
$I_A = m_A K_A^2 = 10 \times 10^3 (2)^2 = 40000$ Kg.m$^2$ (about $G_A$)

Likewise

$I_B = 2 \times 10^3 (0.75^2) = 1125$ Kg m$^2$ (about $G_B$)

$I_C = 1125$ Kg m$^2$ (about $G_C$)

To determine angular acceleration of the rocket, we will apply Newton's 2nd law, for moments about a selected point, we will use center of gravity of the system G, even though can select any other point we want. Note that rotation is about an axis perpendicular to the page.




Newton's 2nd law

$$\sum M_G = I_G \alpha$$

↻ +ve [ you may choose otherwise, but be consistent throughout ]

$$1.5\,T_c - 1.5\,W_c - 1.5\,T_B + 1.5\,W_B = I_G\,\alpha$$

$$\cancel{\alpha = \frac{1.5 \times 8.}{145107}} \qquad \alpha = \frac{1.5}{145107}\left(20\times10^3 - \underset{\times 8.75}{2\times10^3} - 30\times10^3 + \underset{\times 8.75}{2\times10^3}\right)$$

$$\alpha = \frac{1.5\times10^3}{145107}(20-30) = -0.10\ \text{rad/s}^2 \circlearrowright$$

comment! The answer in your book 0.293 is incorrect! I have confirmed with the publisher. The author's original answer is 0.076

In other words it ~~is~~ rotates in the opposite sense that we selected when we set up the equation from Newton's 2nd Law.

We can also analyze the vertical translation of the rocket.

$$\sum F_y = ma_y \qquad \uparrow +ve$$

$$-W_c + T_c - W_A + T_A - W_B + T_B = m\,a_y$$

$$-2\times10^3\times8.75 + 20\times10^3 - 10\times10^3\times8.75 + 150\times10^3 - 2\times10^3\times8.75$$
$$+ 30\times10^3 = (10+2+2)\times10^3\,a_y$$

$$\cancel{a_y = 8.04\ m/s^2}$$

$$a_y = 5.536\ m/s^2$$

Comment: The rocket we analyzed is the Ariane 5, the current premier vehicle of the European Space Agency, launched is from Kourou in French Guyana. Check out their website for excellent technical information. In fact this is how I saw the answer in your book was incorrect after working it 7 times.

Now we have the information for each component. Let us determine the parameters for the whole system.

First lets find center of mass of the whole system. Let us take moments about a ~~vertical~~ horizontal axis through $G_A$

$$m_A y_A + m_B y_B + m_c y_c = m_{total}\,\bar{y}$$

$$2 \times 2\times10^3 \times 6 = (2\times 2\times10^3 + 10\times10^3)\,\bar{y}$$

$$\bar{y} = 1.71 \text{ m} \quad \text{from} \ \text{(South of) } G_A$$

We can now use Parallel Axis Theorem to obtain mass moment of inertia of system about system center of mass.

$$I_{rocket,G} = I_A + I_B + I_c$$

where $I_A$, $I_B$, $I_c$ are about axis through system center of mass $G$.

$$I_{rocket,G} = \underbrace{40\,000}_{I} + \underbrace{10\times10^3}_{m}\underbrace{(1.71)^2}_{d^2} + 2\Big(\underbrace{1125}_{I} + \underbrace{[6-1.71]^2}_{d^2} \times \underbrace{2\times10^3}_{m}\Big)$$

$$= 69241 + 75866$$

$$= 145107 \text{ Kg m}^2$$